The Role of Paradox in the Structure of Fundamental Truth

Douglas Gill

Draft - November 21, 2015

Back of the Title Page

FOREWORD

This paper is, in philosophical terms, an academic discussion on the absolute limit to "fundamental truth". The term "truth" is used in the most general sense. It includes truth by theorem in mathematics, physics and philosophy as well as in conclusions found in societal issues - specifically where there is reference to an appropriate form of "universality". Anything that is "universal" infers there is a reference to an "infinity". On the surface, what appears as a simple truth statement contains hidden "incompleteness" as a natural feature of all references found in the category of "infinity". The topics of this paper are a comparative analysis of the fundamental mechanism behind all such examples. A simple model serves well to demonstrate what we should expect in any structure that takes its reference to an infinity. What appears as fundamental in one framework is always found to exist within a larger and more fundamental framework composed (minimally) as a dualism of elements.

A general principle is validated through the comparative analysis of a wide variety of well-documented subjects. In common terms, infinity is simply defined as something that extends forever, or equally that a given structure displays all applicable self-reference. A new and more analytically correct definition for "infinity" is given. It includes reference to the role of paradox as a general mechanism. For the discussion in topics other than those of pure mathematics and physics, the term "universal" is a more appropriate term than "infinity". However, the same mechanism, for "incompleteness", is found.

It is possible to theoretically represent a state that is infinitely complete for the collection of its elements, albeit without specifically listing all of them. These arguments are "absolute truth statements" since all "truth", for inclusion of the elements, is represented. Cantor's diagonal slash argument is one example. In this case, we find that, from the perspective of a second frame of reference, the truth-statement for the infinite collection is actually false as elements remain unaccounted for. This argument will be discussed in detail.

Such theoretic arguments are well-understood and not in question. Paradox establishes the absolute limit for the inclusion of elements in any "fundamental" truth statement. The general principle for understanding the form of such paradoxical structure is that there is a "flipping" of the relationship for elements that should be common. The result is that two parts are then not members of themselves, even though they remain common in their reference to the larger structure that contains them both. This is a Russell set construction.

Modern science, at the limit of understanding for physics, has the ultimate objective to unify all the forces of Nature into one overriding mathematical principle. In this regard, an important purpose of this writing is to demonstrate that, the very findings derived through science, strongly suggest it is not possible to achieve such an end.

<u>The Theoretic basis of the discussion that follows</u>
An important part of the following discussion is the development of a new understanding of the

relationship of between theoretic, mathematical representation of the universe and the actual basis of its development and form.

The concept to be developed is that the "native" universe is "null-dimensional". Dimensionality must develop across boundaries that represent infinities through a sequential process of self-organization. The mechanism of paradox is responsible for the absolute limit at each boundary. The distinction between this concept and all other theories is that, in such a state, no structure nor organizational laws pre-exist. A consequence of this format is that it is categorically different than can be represented through mathematical formalism. Mathematical descriptions rely on sets of elements and rules that are set down as complete, and no such "completeness" exists.

The process of "self-organization" is well-understood in physical systems, and it is applied as the key principle of development for the universe itself. The native universe self-organizes across boundaries that are proper "infinities". The regions across these boundaries are not "containable" in any single set of rules as they have paradoxical relationship. Why the regions across such boundaries are necessarily paradoxical, and why each boundary represents an infinity, is explained.

The role of paradox is linked to the fundamental process of self-organization through a geometric model. The grandest philosophical framework, for such discussion, is the quest to find a "theory of everything" for the universe. This is a central topic, but the considerations and conclusions extend to all forms in which we seek fundamental truth, knowledge and understanding.

The Theory

A universe that is self-organizing does not obey rules. It sequentially creates them. That process is equally mirrored in all structure and causation within the universe. This is the description of that fundamental framework for self-organization.

What follows is a very detailed attempt to describe what is not describable. The claim made is that the paradoxes we find systemically in countless aspects of observation are not anomalies, but rather point to the fact that the very form the universe takes relies on the fundamental role of paradox.

Any discussion that attempts to legitimize the presence of paradox as a mechanism, in the creation of ordered structure, will not meet the strict paradigm of modern science and logic. This paradigm is that the instance of paradox points to either a faulty premise or incomplete understanding. Consequently, the rational justification that paradox has a fundamental role in the universe is not easily done. The proof can only rely on comparative analysis to identify what is common for its device in each case. If this analysis shows that paradox arises systemically in diverse structures, then the evidence for its role becomes stronger.

The basis of the proof, to be presented, is to show that when the relationship of observables for a

given property of a structure is appropriately flipped it "universally" results in two bases of description that share membership in the original structure but are paradoxically conjoined. The basis of flipping is unique in each case but the mechanism is the same. The claim is made that counter examples are not discoverable, and a provisional proof has been demonstrated. Despite the fact that we see the effect of paradox arising systemically in theoretic and empirical structures, it has always been rejected out-of-hand.. The concept that paradox could be a mechanism of all "form" has never been considered.

If, as claimed, paradox has a systemic role in the universe, it has widely ranging consequences for what we have established as the truth in physics, philosophy and logic. The role that paradox plays, in the creation of fundamental dualisms, will affect the very limit for what we can "know" in any single perspective on what is universal. The relationship across the parts of such dualism (each with its own format of "absolute truth") is paradox. Consequently, each is necessarily incomplete, and the two parts, together, are inconsistent. By definition, there is no way to reduce such relationship to any single comprehensive statement of truth. For one small example taken from outside the realm of pure science, the above paradigm is important to the discussion of the relationship between science and religion - which is one of the grandest of paradoxical perspectives for understanding the fundamental force behind the existence of the universe. If the logic of this writing is correct, there is no singularly fundamental truth to which basis holds the key to understanding the Universe.

One can rightly question whether or not the model in this writing (which has a two-dimensional framework) can possibly point to any fundamental principle on the nature of the universe and is not simply a form of modern numerology. The answer in this interpretation is that it can. Firstly, number systems compress all complexity very successfully by the use of power functions. In this case all dimensionality is compressed linearly across the equality sign. Secondly, in the two-dimensional framework of this thesis, real examples will be given that represent infinities constructed and expressed in two dimensions. The claim is that an infinity in a dimensionally simple structure has the same fundamental properties as an infinity at the limit of dimensionality.

The conclusions of this paper are extended to all frameworks in which we observe the universe and its parts and to all forms in which we seek fundamental truth and meaning.

The Wrong Question in Physics We Have Been Asking Is: what is wrong with our current view of the universe?

The Correct Question Is: why are there two absolutely correct and internally final views of the universe that are paradoxical?

> *No scientific theory or principle proves paradox to be an anomaly. To claim that it is, in spite of overwhelming evidence to the contrary, is a philosophical bias. The role of paradox is examined through the footprints it leaves and, in each, a common element of structure.*

The Footprints

The presence of paradox in all that we observe is the mirrored footprint of immaculate non-order.

A General Principle on the Role of Paradox

The study of paradox, in relatively simple theoretic and empirical structures, demonstrates that the presence of paradox is far from an anomaly to be explained away by better understanding. Paradox arises whenever we, in any manner, create a framework that takes reference to infinity as a boundary. When this occurs, the structure, in one of two contexts, spontaneously defaults to a dualism. The two parts of this dualism refer to the same property of the structure under consideration, but are not rationally contained as a collection of common elements. Paradox is the mechanism behind this default to a dualism.

This calls into question important issues. What is the nature of infinity that is tied to paradox? Is there a general principle for how the mechanism of paradox works? If paradox is a general mechanism in the universe, what is its impact on our understanding of the universe? These are the topics covered in this writing. The conclusion examined is that paradox is an inherent, fundamental feature of the universe itself, as well as everything we experience within it in physical terms and that we consider in theoretic terms.

One of the topics that naturally arises, for the study of infinity and paradox, is the quest, in physics, to find a "theory of everything". In the most general sense, this theory must be all-encompassing for the description of an infinity (the universe itself). If it is not complete (does not account for some portion of the infinity) or is inconsistent (inconsistent within the framework of some competing theory) then we have not described everything. In this regard, paradox may prevent us from ever arriving at a final, fundamental understanding of the universe. Our current understanding appears to lead to this very conclusion.

If paradox is, indeed, a fundamental feature of Nature then its property will apply to the universe itself. As such and by definition, the universe, in its "native" state, would not contain any rules of relationship that could be the basis of its structure. This universe would not have structure in the sense of any reality, and in simple terms, it would not "exist". It would be best described as an "immaculately non-ordered" "null-state", and the mechanism of paradox would be responsible.

Surprisingly, there is a rationale in which a "real" universe can still arise out of such "non-order" and "non-existence", and the basis of this "self-emergence" can be understood in human terms. A relatively simple geometric and mathematical model, in a two-dimensional framework, can be used to demonstrate the mechanism behind the force and the process of "self-organization" that arises in an immaculately non-ordered state. This mechanism does not require the pre-existence of any laws of order or even the existence of a universe in the largest sense, and it actually normalizes the relationship of the parts necessary to have a real universe. Of final importance, this two-dimensional model, in spite of its simplicity, has direct application to that which is observed in the real universe. A clear and large body of evidence for this model will be presented in both theoretic principles and empirical structures of Nature.

The Model:

There are two contexts to the development of the model. The first is to show how the mechanism of paradox works in individual examples (both theoretic and empirical), and the second is to build a general model that goes to the very heart of how the universe fundamentally incorporates paradox in its structure. The unlikely candidate of Russell's paradox, provides the linguistic framework for this model.

Russell's paradox refers to a "universal" set that collects all sets of objects with the property that the members are not members of a stated property. For example all objects that are not blue would be a listing found in this master set. The master set for all such items is also "not a member of itself". This is necessarily the case because the master set must also share the property of all the elements within it. The question arises whether or not this universal set (also here referred to as a "not-set") should be placed in its own listing. If it is placed within, then it shares membership in itself which is a contradiction to its own property. If it is not placed in itself, then it meets the condition for "property" that it is not a member of the listing of all sets that do not share property. However, equally we find contradiction that it should be placed within itself. Of course, all of this is completely self-circular and without resolution.

There is no resolution to the dilemma for how to place the Russell set to itself other that to deny the legitimacy of the argument in the first place. Indeed, this is the generally agreed upon solution. However, if it can be demonstrated that the same construction is found systemically in Nature, then it would be wise to look deeper into the possible role of paradox as a mechanism rather than to discount it.

A key element, in understanding how paradox is created across fundament collections of parts that also take their reference within a larger structure, is to redefine what constitutes an "infinity". Infinities necessarily contain the strong element of "self-reference" that is also found in Russell's paradox. Specifically, although theoretic examples of infinitudes can be formulated, closer examination reveals that, the structures so created always leave out part of their membership. This contradiction returns our attention to Russell's paradox for how a "universal set" for any "property" is to be established.

On the grand scale, for discovery of a theory of everything for the universe, we presently have two completely successful theories that are paradoxical to each other. Are we ignoring what Nature is telling us about the role of paradox in the universe? If all that we observe within the universe has an element of paradox attached, then the universe should have this property, and paradox itself must be a mechanism of creation.

The case will be presented that when we attempt to form singular understanding and acquire knowledge, in any context of universality, the framework so formed always defaults to a dualism of categorically paradoxical, yet correlated, elements. There are very simple examples of this that can be use as a guide in deciphering what goes on in the most complex of examples. The difficulty in understanding how this works is not the complexity of the arguments but rather the

ability to "stand back" and accept what cannot be known for the mechanism at work.

Paradox is the mechanism that causes singular truth to spontaneously break apart into a fundamental dualism of elements. No further simplification of structure is possible. This is in spite of the fact that the parts, so found, remain bound, as common, for their shared property. This form of dualism does not bend to our logic and resolve to any single unified realm of rationally connected elements. Paradox operates to categorically deny such resolution. Something will always be left out. The two sides to this are "incompleteness" and "inconsistency", as well as the "entanglement" of these two frameworks.

In academic terms, the sophistication of our theories and tools have allowed us to reach to the very limit of infinity in its many forms. The serious problem with presenting a study of the limitations to such understanding is that our innate desire to seek fundamental truth always gets in the way. In spite of the overwhelming evidence to the contrary, we would rather continue the quest for fundamental truth than admit the limitations to finding it.

Understanding Counterintuitive, Paradoxical Structures

There is no direct way to rationally interpret the relationship of elements in fundamentally paradoxical structures. The best we can do is examine and define the *footprint* of the mechanism of paradox that applies universally for all such examples. This pattern, which is not directly intuitive, still becomes clear. What emerges on a grand scale in each case, is an understanding of the systemic role of paradox at all levels of complexity within the universe and for the universe itself as the mechanism of "immaculate non-order". Each example is a "clue" to this general form.

Note: in what follows, the term "observer" and "observer-position" is used extensively. This is a term used in mixed contexts for EPR phenomena. In some interpretations it is taken to be an active force by a human participant that creates classical reality. In this writing, the term "observer" simply refers to the framework in which observation is supported. It is a category for the specific dimensional level that allows observation. This is the "classical" framework, in which space and time have discrete (and paradoxical) identities.

A Universe in Self-contradiction Between Existence and Non-existence

Working from the initial premise that paradox is a fundamental feature, of the universe (to be justified in sections that follow), we can speculate on what this universe will look like. Its singular "native" state, allows the display two parts as long as they are paradoxical to each other. In this case, the parts have no capacity to display common form - the state and the relationship of its parts are immaculately non-ordered, and no "common state of existence" applies across them.

Regardless of this prohibition, the fact that two paradoxical frameworks exist means that each can have its own boundary for meaning, (together displaying no meaning). Each perspective has its own structure of "horizon" for an infinity. The two frameworks of Gödel's incompleteness theorems (that universal structures are necessarily either "inconsistent" or "incomplete") have

application. One of the parts will take the property of internal inconsistency to our ability to draw conclusion, and this is exactly what we find in quantum structures that are non-collapsed. The other part will take the property of incompleteness, and this is exactly what we find in classical structures (incompleteness in time and physical boundary). Furthermore, regardless of how we attempt to resolve the frameworks of these two parts, our efforts will fail. Paradoxical circularity prevents it.

In philosophical terms, paradoxical circularity has historically been regarded as anomaly that begs for discovery of some formal argument for its handling. Great efforts have been directed to finding a logical resolution on how the properties of infinity, identified above, can be integrated into a single, description in logic and mathematics. However, it may be the case that such circularity is a natural feature of the universe that has never been assessed. It is the central point examined in this writing. Non-resolvable paradox is described as the source of all complexity that allows the existence of an observable universe to arise in the first place.

A force and trajectory for self-organization will be identified. On this issue, there are two perspectives. The following descriptions are presented as a guide to the framework that will be justified throughout the sections that follow.

How Prohibition Works in Fundamentally Paradoxical Structure

Infinitudes come in many forms but all of them represent one of the two formats of paradoxical construction. These states are either static (quantum-like) or dynamic (classical-like).

For the Elements of Stationary (Static) Structures: Analysis can be done on a case by case basis (for correlated dualisms) to show how the relationship of elements is "flipped". In so doing, these elements develop paradoxical identities, yet remain bound within the original state. Consequently, the structure is rendered closed to observation from the outside - the outside being any form of observation on the interior. Each example has its own unique signature that, nevertheless, points to a general principle.

For Dynamic (Non-Settling) Structures: Dimensionality has a key role in creating boundaries that are infinite. Dimensional levels must "develop" sequentially, and the structures across these separate "platforms" do not share common properties. Non-resolvable paradox produces a force and trajectory referred to in this writing as "dimensional bifurcation". Continuous change results, and dimensional complexity builds. Elements are "subsumed" (left behind - as closed) by hiding them "at lower dimensionality" in the ongoing process. The regions that have been subsumed are internally "closed" to observation of their parts.

The accumulation of such structure is mathematically normalised through power operations which bring the overall structure across these boundaries down to a common plane across the equality sign. This means that process of dimensional accumulation is fundamentally different from its representation through formal mathematics. The process of bifurcation occurs from a state that displays no form and, in contrast, mathematical systems contain all appropriate form

(elements and operations) as pre-existent. This is the reason that the description of the universe, through mathematics, is fundamentally different from the process of its creation and form.

The above two frames of reference (static and dynamic) are the paradoxical dualism to the existence and observation of "infinite boundary". To find these two features in a wide variety of examples (both static and dynamic) supports the contention that a systemic mechanism is at work.

Conclusion:
The fundamental and systemic role of paradox in the universe is only observable through examination of its "*mirrored footprints*". Beyond the analysis of simple dualisms this pattern is expected to be the basis for all the complex structure of physical reality.

The Footprints

The presence of "paradox" or its partner "change without boundary" in all that we observe is the mirrored footprint of immaculate non-order.

The Relationship of Quantum and Classical Structures:

Information:
Paradox is the fundamental mechanism for "prohibition to information"across that common parts of any structure. Simply put, if a cohesive structure is opened to observation, of two parts (as a minimum), and they are found to be paradoxical to each other, then we have been prohibited from information on the relationship of the parts even though the structure remains cohesively formed. The same principle will apply whether paradox is displayed in the form of a simple dualism or as the entanglement of more complex parts.

$i = (\sqrt{-1})$:

Distributive (classical) logic:
In the logic of a classical description, forming the square and taking the square root are reversed, commutative functions. In other words, if we are permitted to take the square root then equally something has been squared.
The squaring operation, 1^2 has the form (+1)(+1) = +1 or (-1)(-1) = +1. In this breakdown of the parts, both elements in a single squaring operation are seen to display the same sign, either plus or minus. Conversely, taking the square root is necessarily an operation on +1 (since this is the only possible result of the squaring operation). Accordingly, this trivially produces the two possible alternative roots of +1 or -1. The knowledge on which result applies does not exist.

Non-distributive (quantum) logic
The identity of i^2 is -1, which has no meaning in classical terms. This is because the plus and minus signs are entangled in the quantum construction under the operator "and" when, classically, they should be discrete under the operator "or".

Going back to the classical framework for how the squaring operation works, that which has been previously squared (and is contained within the square root function) is the value -1 (not +1 as it should be in classical terms). The classical alternatives of the possible roots (plus or minus signs) have been entangled in the quantum construction under the operator "and". The alternatives are no longer discrete.

The validity of the construction for *i* is not in question as it is the basis of all that is quantum-mechanical. The important point is that the quantity *i* "entangles" two classical operations that should be discrete across the operator or. Once *i* has been incorporated into any mathematical construction it takes on a non-classical property, and observation and description in the framework of classical mechanics is prohibited. The term "entangled" means that the parts of a structure, which would otherwise have discrete identities in a classical description, are "dispersed non-discretely" throughout the entire "space". In simple terms, it is not possible to point to individual locations for the parts that define the structure.

Thus, we find that, for the description of the universe and its parts, two fundamentally paradoxical constructions exist. One is quantum-mechanically based and the other is classically based, and they are both equally valid within their own framework. This mathematical relationship is the fundamental *footprint* we have to the most general property of the universe - that it contains a dualism of non-common (paradoxical) elements that are, nevertheless, members of the same state.

EPR Experiments

Both classical and quantum-mechanical descriptions apply to the same "event-spaces" of EPR (Einstein, Podolsky and Rosen) structures. The factor *i*, is the key element of the relationship between the two frameworks and consequently they are necessarily paradoxical. There is no point looking for resolution to this paradox as the presence of the factor *i* in the quantum-mechanical half of the description makes this impossible. The last point, in this introduction to EPR phenomena is that the presence of paradox, incorporated by inclusion of *i* in the quantum version of the same space, in no way directly infers that the alternative classical description is somehow wrong.

EPR experiments provide an excellent point for analysis of the role of paradox that extends to the larger framework of the universe itself. The observer is able to select, by manipulation in the framework of a given EPR apparatus, whether the event-structure is classical or quantum-based. By definition, we have discovered a dualism of descriptions - one classical and the other quantum-mechanical, for a single state, across which a format of paradox applies.

We see mirroring of the fundamental framework of the universe, described above, in the section, A Universe in Self-contradiction Between Existence and Non-existence. The factor *i* is the mechanism for the prohibition to information across two parts. As described, this prohibition is also the device that allows information and order to arise in each part, in the first place, by the creation of two fundamentally distinct regions paradoxically conjoined.

The mechanism that places the classical and quantum forms in paradoxical relationship can be clearly identified. It is a dimensional boundary. The two spaces across this dimensional boundary have categorically distinct and non-reducible dimensional platforms, and, categorically, a single form for description cannot exist. In mathematical terms, the operative mechanism creating this boundary is application of the factor *i* to the quantum version of the space.

In mathematical terms, the transformation to the classical version of the same space is represented by taking the square of the modulus. This fundamentally changes the dimensional nature of the structure and rectifies the space to the classical format of the dualism.

1. Half-silvered Mirror Experiment

The half-silvered mirror experiment is a construction of two Mach-Zehnder interferometers in series. (illustrated in Figure 1). The first device "splits" each photon in a beam of photons and the second device rejoins each photon. The photons become entangled across two paths. Internally to the structure, these paths include reflecting mirrors that separately redirect the two entangled paths back toward each other so that they are rejoined at the second half-silvered mirror.

Classical Format

Each photon is a classically fundamental particle. This means that it is not possible to "divide" a photon into smaller classically identifiable parts. We are categorically not permitted to observe half a photon in the way we are able to observe half an apple. Consequently, if photons were responding as "classical particles" at the first half-mirror (position *I*), then each would have to choose one of the two available paths. We do find this to be the case when some form of observation (which includes obstruction by an observer) occurs within the "space" of the apparatus. The photon will always be "known" to either "exist" or "not exist" relative to each path from the first half-mirror.

Quantum Format

The classical basis is only one-half of the paradoxical framework to be discovered in this EPR experiment. The photon will traverse the two-path structure in an entangled format (simultaneously as a waveform) as long as no observation or disturbance on the path-structure occurs. This process has a non-classical, linear description that mathematically requires the factor *i* be applied to the entangled path structure at each 90 degree turn. This makes sense, from the outset, since we have seen, above, that the factor *i* is the fundamental identity that transforms a classical relationship of parts into an entangled state having no classical meaning.

At any location during the period that the photon is in mid-flight across the apparatus (in its entangle form) it can be spontaneously raised to the classical level by interjecting either a blockage or disturbance to one of the two initial paths. The photon then "collapses" instantaneously to a classical basis in which it is found in whole on one of the two potential paths. It is no longer entangled across two paths.

This instantaneous action violates the limit of the speed of light for communication of

information across the two paths because there is no know basis of communication that would mathematically describe how the photon "knows" (has information) that it should be on one of the paths and not the other. Of note, there is no restriction on the distance of separation between the two paths for this transformation. When the photon instantaneously collapses knowledge has been created to the location of the photon that did not pre-exist, and the relationship between the quantum and classical descriptions for the same event is paradoxical.

On the one hand, the paths are indistinct and linked without separation in space and time (the quantum, entangled version), and on the other hand, the paths are indeed distinct and separated in space and time (the classical version).

The standard, formal interpretation of this is that the presence of paradox is necessarily an anomaly indicating there is something "unforseen" going on which is not accounted for in our current mathematical understanding. Either the classical or the quantum-mechanical description must be wrong, or they are both wrong. However, in the model adopted in this writing, the assumption is that paradox is not an anomaly. The simple interpretation that there is something wrong with one or both of the two descriptions misses the point that paradox is a fundamental mechanism - as well as the most general feature for the construction of the universe.

The evolution represented in the transformation between the quantum and classical bases is across two distinct dimensional levels. The classical level has a higher (more complex) dimensional construction that allows "disentanglement" of the imaginary direction in space (at 90 degrees) associated with the factor *i*. The imaginary direction transforms to the equally imaginary property of sequencing in "time".

The universe, in the largest sense, and the very small space of the half-silvered mirror experiment share the same characteristic that they are fundamental spaces. We find that a "prohibition to information" through the presence of paradox applies for both. This prohibition is the mirrored *footprint* on the nature of the universe itself - that it is immaculately non-ordered. In this arrangement no single final truth exists for the property of the state. Rather a dualism applies across which information is prohibited. Such structures are null-states for information because, in any single view, the information we are allowed is either inconsistent or incomplete. This arrangement mirrors and thus conserves the fundamental structure of the universe itself.

The operative mechanism of paradox that prohibits the common elements of a given structure from exhibiting common membership is easily analyzed. This mechanism is that a "flipping" of context occurs for the relationship of the parts of the overall structure. This forms the parts as not members of themselves. The basis of "flipping" is unique in each case, but the general mechanism is the same. It deserves repeating that this is the framework of Russell's paradox.

next page

Detailed Geometric and Mathematical Description

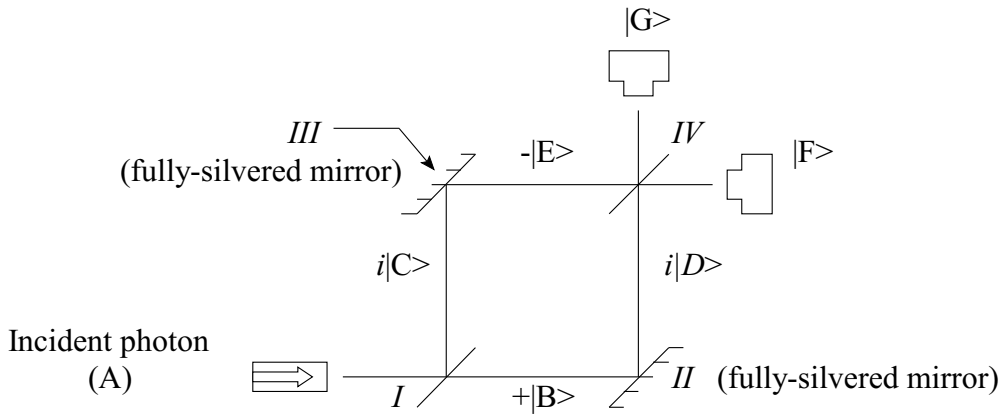


Figure 1. The half-silvered mirror experiment. - Half-silvered mirrors are found at *I* and *IV*. Fully-silvered mirrors are at *II* and *III*. This structure is called a *space*. The simultaneous "evolution" of the quantum version of the state for the two paths is shown including the signs that apply where *ii* |E> = -1|E>, (see below).

Some paths are "real" (not having *i* attached), and some are imaginary. The real paths |B> and |E> have reversed signs to their direction across the apparatus, which is another perspective on the flipping action that transforms a single structure between two paradoxical formats. The effect of the interferometer at *I*, is that the dimensional framework of the overall structure cannot support an independent direction for time as found in classical space. This dimensional reduction is displayed by the angularity between the real directions (those not having *i* attached or where it cancels). The angularity is 180 degrees not 90, and to each of these real directions, the classically orthogonal direction, at 90 degrees, is now imaginary. Because space and time cannot take on separate discrete properties, two paradoxically separate descriptions for the same space apply.

$$\text{If } a > 0, \quad \sqrt{-a} = i\sqrt{a}; \quad i^2 = -1 \tag{1.1}$$

$$A \rightsquigarrow |B\rangle + i|C\rangle \tag{1.2}$$

$$|B\rangle + i|C\rangle \rightsquigarrow i|D\rangle + i\,i\,|E\rangle \qquad (\text{note: } i \times i = -1) \tag{1.3}$$

$$i|D\rangle - |E\rangle \rightsquigarrow [\, i|G\rangle - |F\rangle\,] + [\, -i|G\rangle - |F\rangle\,] \tag{1.4}$$

$$= (-2|F\rangle) + i(|G\rangle - |G\rangle) \tag{1.5}$$

$$= 2(-|F\rangle) \tag{1.6}$$

The unitary evolution of the state:

The "unitary" (simultaneous) evolution of the quantum mechanical state, in which paths are entangled, is indicated using ket notation in Equations (1.2) to (1.6). There is no clear sense of direction for the passage of the photon across the structure, in a classical sense, even though it crosses and exits the apparatus. From a classical perspective, we cannot even state that time flows because if time has flowed, the particle would have moved in a consistent direction. Time

is transformed, by down-conversion, into one of the orthogonal directions of the space. We can pick out a vague commonality to our classical sense of what time represents. In its classical form, time still accumulates as "imaginary".

Thus, entanglement produces two formats for hiding the distinction of path across the apparatus. Paths are either imaginary (when *i* is attached) or cancel in aggregate for their direction across the structure because their signs are reversed.

When the description for the state is quantum-based, the photon is always recorded on the route |F>, after the second interferometer *IV*, This is mathematically represented as the state $-2|F\rangle$. The path structure has been properly recombined, at the classical level, without any form of disturbance within the space between the interferometers. In other words, the classical observer has not forced the photon to display a higher level of dimensionality within a space that has a quantum signature.

The factor of 2 for the ket vector (-|F>), in (1.6), arises because this is a linear expression of two paths in parallel. Each part contains one entangled half of the classical structure of the photon. It takes two of these sub-classical paths to represent the photon on a singular path after the second interferometer. Of note, if this path structure were classically based, the linearity of path would be in series of time, not parallel, for a complete delineation of potential paths.

The Classical Collapse

When the state of the half-silvered mirror experiment represented by passage of photons, evolves under classical rules, the full event structure requires a sequencing in time for the display of its full potential. This is opposed to the unitary evolution of the quantum version of the state in which the full potential is displayed unitarily along the paths. In the classical framework, the "capacity for dynamic change" requiring sequencing in time is allowed.

Once the interior path-structure has been classically disturbed, the squared modulus rule (the square formed as a positive number) is applied to the linear path-structure to establish probability across them. For the separate paths represented as [path *I* : path *II*] is mathematically $[\ |i|^2 : |-1|^2\ ] = [1:1]$. Squaring of the terms *i* and -1 raises each path to the classical level, and there is then an equal probability to find the photon on either path.

2. Bell's Inequality

Bell's Inequality is universally regarded as the proof that, the classical description of the universe is necessarily "somehow wrong". The experimental demonstration of Bell's inequality consists of sending two classical-level particles entangled, for a specific common property, at the quantum-level, off in separate directions. These particles are classically distinct for their locations in space and time and should display properties that have independent values (not entangled). However, it is found that the observable states (the values found on each path) for the given property of the experiment are entangled. They obey quantum rules and are not independent even though the particles have separate locations in the classical state.

There is an important contrast here to the half-silvered mirror experiment. In that case entanglement was created for each single photon, by sending it simultaneously along a two-path structure. For Bell's inequality, entanglement is displayed for two classical-level particles that are sent on separated paths.

Thus Bell's inequality, using a two-particle system, sets the classical and quantum descriptions at odds simultaneously (in parallel) in the same experimental setup. It is not an either/or situation, based on whether or not observation has occurred within the path structure, as in the half-silvered mirror experiment. In that case, at the classical level, there was a statistical series of possible values in time. By contrast, in Bell's inequality, the two particles are always classically separate particles on separate classical paths, and they are also simultaneously entangled for the value they each display.

Thus, in agreement with what quantum theory predicts, it is found that the classical description fails. Specifically, the quantum signature of the relationship of the two particles does not collapse when observation occurs (it remains "robust"). Consequently, the universally accepted interpretation is that, regardless of any future refinement of theory, our classical description of reality must be somehow wrong.

However, there is an important error in the interpretation of the results for Bell's inequality. If Bell's inequality were based on dispersal of one fundamental particle onto two separate paths (as in the half-silvered-mirror experiment), then we would correctly expect the two-path structure for the particle to collapse. On collapse, the dimensional level of the system would display only classical features. However, this is not the case. Because a two-particle construction exists, the quantum and a classical structures have been placed in coincident, parallel superposition. The question remains whether such an arrangement is possible and should not be rejected, out-of-hand, without consideration. If paradoxical dualisms are a fundamental feature of the universe, then Bell's inequality is telling us something entirely different than currently understood.

If there is a mechanism by which correlated quantum and classical environments can exist in parallel, given the role of paradox, then it would be possible to see both classical discreteness and quantum superposition displayed together simultaneously. In this case the quantum structure would be robust even in a classical environment.

As for all structures that display a paradoxical dualism, and represented in each of the above experiments, the quantum side of the structure is created by a specific flipping of relationship. In the case of Bell`s inequality, the relationship of two particles (that are independent for their property of spin, in classical terms) has been manipulated (flipped) by a device such that they display concatenated spins. This occurs because the particles are emitted simultaneously from the same source.

The flipping of relationship is further represented by the fact that the two-particle system displays both quantum and classical signatures. The structure of the relationship of the particles is always

quantum because the spins are always entangled, and the structure is always classical because the two superposed spins must always collapse to a single classical value when either of the particles is observed.

Comparison to the Half-silvered Mirror Experiment

The relationship of the observer to concatenated quantum and classical states can understood through what is found in the half-silvered mirror experiment. In that case, the observer's position is always classically based relative to any disturbance of the quantum-level structure. Consequently, when the observer disrupts the quantum-level system of paths, they must take on a classical structure. As described above, this is a dimensional transformation for the path structure in which the directions in space and time become disentangled. The transformation across the two formats, for the observer, is an either/or relationship in series.

For Bell's inequality the relationship of the observer to the system has changed. On the one hand, the observer is positioned outside the quantum-level basis because the spins of the particles are entangled, and remain so; however on the other hand, at the classical level the spins of the particles must collapse when observed. Consequently both the classical and quantum structures are robust in parallel relationship to the position of the observer.

Detailed Mathematical Description for Bell's Inequality

An example of Bell's inequality employs photons that are entangled for their polarization attribute. They have the same polarization as long as the measurement device measuring each photon has the same angle. A calcite crystal is located on each path of the two-path structure. When one of the calcite crystals is rotated it will introduce an error for the agreement of spin with the other photon. The two-particle system has been collapsed by classical interference since we have knowledge of the value of the spin of one of the particles. If one crystal is rotated +30 degrees, theory and practise show that an error in agreement across the two possible values of polarity is 1 in 4. Then if the other classically distinct photon is rotated by the -30 degrees it will also contribute a separate (local) error rate of 1 in 4.

Classical statistics specify that the error rate for this system of two photons, with angularity of 60 degrees across each other, is determined by addition of each of the error rates minus a factor of chance coincidence that they accidentally take the same value.
The error rate is randomly (1/4) + (1/4) - (chance factor of agreement) = 1/2 or less

However, under quantum theory (appropriate when two particles are entangled) the error rate is calculated as $\cos^2(60) = 3/4$, and this is at odds with the error rate using classical criteria. In agreement with the interpretation and calculations by Bell, it was found, in an experiment conducted by John Clauser, that the correct error rate is based on the quantum-mechanical not the classical calculation. Thus, the conclusion is that reality is not classically based. It must be "nonlocal" (quantum). The error in this interpretation has been discussed above.

Bell's Inequality in the Framework of the Model

To justify Bell's inequality as one example of the theory that paradox has a systemic role in the universe, it must be shown that a flipping of observables has occurred for structure by which a paradoxical relationship is created between two descriptions for the same structure. In this case the structure is the relationship between two photons for their spin property. On measurement in a classical basis, these spins are independent as determined by classical probability.

The mechanism of "flipping" for this classical relationship is that two photons are emitted from a device that entangles their spins as a unitary state. As defined in the model of this writing, two descriptions are created that apply to the same structure but that are paradoxical. On the classical side, the property of the photons for the observable of spin displays classical separation in space and time. However, on the quantum side the same two photons display a unitary (quantum-mechanical) identity for their separate spins.

3. Hardy's Paradox

Hardy's paradox is not simply about particles being measured in an EPR device. It is about particles being simultaneously measured and not measured across the intersection of two EPR devices - the waveform of all possibilities. The event-structure is recorded based on the condition that both particles reach the exit ports that would necessarily remain dark in classical terms. This represents a specific, classically observed effect of the waveform. The paths contributing to outcome are found to be entirely counterfactual, in classical terms.

The conclusion to be presented in this writing is that certain discrepancies between theoretic calculations, and experimental results for the experiment are not factors of experimental error as suggested by experimenters. Rather, these discrepancies are the signature of a causal mechanism, and the divergence between the theoretical and experimental results points to a new principle.

Note on the terms: Interaction Free Measurement (IFM) is a technique in which particles, at the quantum level, measure each other instead of being measured by a classical mechanism. The result is that the system does not collapse as would be the case for any classical measurement technique.
Weak Measurement is the technique of recording the results of IFM using statistical methods that disturb the system only weakly. The pattern of paths taken, in quantum terms (their history), can be collected for analysis.

- Hardy's paradox was first proposed as a "thought experiment" by Lucien Hardy.

- The mathematical analysis for what happens in Hardy's paradox was then presented in the paper, The Theoretic Analysis: Revisiting Hardy's Paradox: Counterfactual Statements, Real Measurements, Entanglement and Weak Values. Yakir Aharonov, Alonso Botero, Sandu Popescu, Benni Reznik, Jeff Tollaksen. It is found at, http://arxiv.org/abs/quant-ph/0104062.

- Finally, an experiment that demonstrates Hardy's paradox was conducted by Jeff Lundeen and Aephraim Steinberg. It is found at http://www.photonicquantum.info/Research/publications/Lundeen%20PRL%20102,%20020404%20%282009%29.pdf. It was also presented in a lecture by Professor Steinberg at the Perimeter Institute (PI). This is found at: http://www.perimeterinstitute.ca/videos/praise-weakness

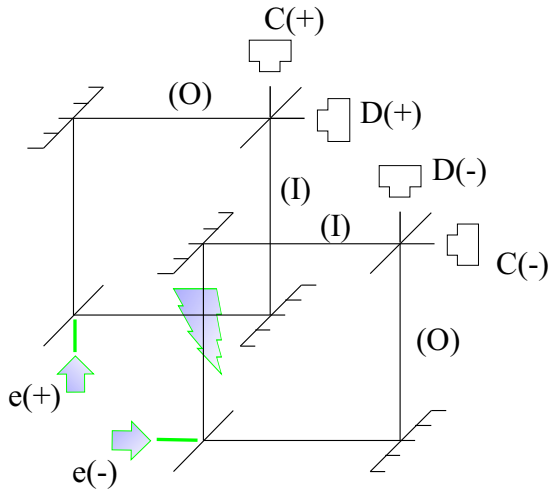


Figure 2. Hardy's Paradox - Two concatenated (overlapping and entangled), EPR structures, each similar to the half-silvered mirror apparatus, have inner (I) and outer (O) paths. The interference in each structure overlaps on the inner paths at the position of the lightning bolt figure. The ports labelled (D) would normally be dark ports if the system were is not disrupted, as is the case of the simpler half-silvered mirror experiment. The difference in the setup is that particles across the two halves are both disrupted and not disrupted simultaneously. The particles in each interferometer are anti-particles to each other as indicated by e(+) and e(-). The disruption/non-disruption that occurs owes to the fact that, rather than a classical-level observer doing the observation, it is each particle "measures" that the other. Thus the overall structure remains quantum-based (i.e. no observer has enter the quantum-level space to make a measurement except on exit at the ports D(+) and D(-). A pattern results in which all paths are found to be entangled for the firing of ports D(+) and D(-). No classical intervention was required for paths that would normally not contribute, to contribute to the event structure. A weak measurement technique is used to statistically collect the sequence of path-structure causing D(+) and D(-) fire simultaneously.

Hardy's Paradox takes the category of EPR experiments to a new level of complexity through the unique concept of concatenating (linking) and thus entangling the two waveforms of fundamental particles on the overlapping paths of separate interferometer structures such that each path-set interferes with the other. The geometric configuration of each set is as found in the half-silvered mirror experiment. The overall structure is shown in Figure 2.

When one of the particles is on the inner path of its structure (the first structure), it overlaps the paths of the second structure. This means that its presence constitutes a disturbance (measurement) of the system of the other particle in the second structure. If we consider this in the framework that would normally apply for classical measurement of a quantum system, the presence of the particle would cause the collapse of the second structure to a classical basis, and the particle in the second structure would display distribution across both ports (C) and (D). This is as was the case in the half-silvered mirror experiment when the path-structure was disturbed. Of course the same situation applies if the particle in the second structure is on its inner path. In both cases, the systems will display, a classical-level distribution across their exit ports, (C) and (D).

However, because the particles themselves are causing the disturbance (not a source of disturbance from "outside" of the overlapping quantum-level structures), called "interaction-free measurement", the effect of measurement has an entirely new framework of causation. This event-structure is measured specifically from the classical perspective when the exit ports (D), in each structure fire simultaneously.

The particles chosen for the experiment are important. Each is an anti-particle to the other, which means that if they meet on the two inner paths, in classical terms, they should destroy each other. However, this is not the case at the quantum-level of the interaction between the two particles. This must be the case since it is the simultaneous firing of ports D(+) and D(-) that is measured - meaning that both particles had to be on the inner paths of their respective structure. The particle description fails to account for the fact that the waveform does not obey classical rules.

The Strange Situation When Both D(+) and D(-) Fire

The causal structure, that detectors D(+) and D(-) fire simultaneously, which is both predicted by quantum theory and confirmed in experiment, is counterfactual in classical terms.

The following is a quote from the paper by Aharonov (found on page 5). Aharonov lists the counterfactual framework for paths taken that apply for the simultaneous firing of D(+) and D(-), when it is measured at the classical-level at the point of exit from the system. Note: (bracketed numbers added).

"Let us return now to Hardy's example. As we will show, the complete description of what occurs is encapsulated in the three basics counterfactual statements which define the paradox:

(1) • The electron is always in the overlapping arm.
(2) • The positron is always in the overlapping arm.
(3) • The electron and the positron are never both of them in the overlapping arms."

We can conclude, from the above, that the causation of the simultaneous firing of ports D(+) and D(-) does not depend on any classical framework of the paths taken by particles.

The Two-photon Absorption Switch

The experiment by Lundeen and Steinberg is conducted on photons not electrons and positrons. Pairs of photons are produced that are entangled for their polarizations. A two-photon absorption switch replicates the destruction that would occur when an electron (e-) and a positron (e+) meet on the common inner paths of the two devices in the electron/positron version of Hardy's paradox.

Lundeen and Steinberg, state that the difference between the pure and experimental results is because of two factors. The first is that the two-photon switch is "inefficient" and removes only some of the paired photons (85%). This means that 15% of the pairs survive and are passed on to the detectors. The second is what is described as the interaction-free measurement (IFM) probabilities. Under this writing, only the IFM probabilities are responsible for any discrepancies from pure quantum theory. The two-photon absorption switch does not contribute to error.

If the two-photon switch can be justified as not contributing to the error rate (a means for doing this is described below), then the discrepancy between the pure and experimental results must point to some new and undiscovered principle. That principle is contained in the geometric model presented in this writing.

A reference on the nature of the switch inefficiency is found at:
http://www.rp-photonics.com/two_photon_absorption.html
Quoting: "Two-photon absorption is a process where two photons are absorbed simultaneously, exciting e.g. an atom or ion to a higher-lying state, with the energy increase being equal to the sum of the photon energies. This is a nonlinear process, occurring with significant rates only at high optical intensities, because the absorption coefficient is proportional to the optical intensity..."

Why inefficiency of the switch is not a factor of error.

A dualism of elements that do not share common property yet are members of a common "domain" or "set" form a not-set or Russell set. In Hardy's paradox the waveform of all possibilities that cause the simultaneous firing of the ports D(+) and D(-) takes this form. Based on the results of the experiment (e.g. see quote from Aharonov's paper, above), it is clear that we cannot state that any of the single components of the waveform (it history) is responsible for the outcome observed.

Note: it is entirely incorrect to refer to particles as actually existing on any of the paths within the entangle interferometers at any time. The paths form a complex waveform of possibility that is not collapsed along the various paths. We can only refer to the existence of observable properties for the particles when they are classically recorded which is at the point of exit at D(+) and D(-)

Specifically the waveform represents the contributions of all the paths (not just some) to the outcome. Furthermore the outcome that is register is the "pure consequence" of what is not possible, in classical terms. We are not registering any of the history that applies if, for example,

C(+) and D(-) were to be recorded.

In a classical basis, "real" particles (real meaning, particles that are finitely observable) would cross the structure on "real", separate paths and those particles would never arrive at D(+) and D(-). In the non-classical framework, for this experiment, the property of particles examined is that they simultaneously fire ports D(+) and D(-). This outcome is distributed probabilistically in the history of all contributing paths. In a classical understanding of the experiment, there is no context that the particles could measure each other and survive - they would have both been destroyed! However, to repeat, since the basis of the interior of the concatenated paths is entirely at the quantum-level, "destruction of the particles" has no meaning. That the particles simultaneously exit at D(+) and D(-) is the consequence of all possible histories, rather than that they would be know to have talk paths that cause their mutual destruction. This is evident from analysis of history as follows:

A) the dark ports fire simultaneously when the particles are both on the inner paths and have been destroyed, and
B) the dark ports fire simultaneously when the particles are both on outer paths.
This is the waveform of all possibility that is the defining feature of quantum structure. A and B do not share common property for the simultaneous firing of the ports D(+) and D(-), and they are not common elements in the set of all conditions that cause the simultaneous firing of the dark ports. However, indeed, they do form a set of common elements in the property that ports D(+) and D(-) fire simultaneously.

When photon-pairs are not destroyed because of the inefficiency of the two-photon absorption switch, errors are introduced to the waveform of simultaneous firing of the ports D(+) and D(-). The photon pairs that survive contribution when they should not. However, as will be seen, this contribution is dispersed equally to all parts of the waveform as follows.

For condition A, an excess of information exists when paths contribute that should have had particles destroyed on them, but they are not destroyed.
For condition B, an equal excess of information exists when paths for the particles-pairs, not destroyed on the inner paths, contribute to outer paths in the simultaneous firing of D(+) and D(-).
The excess of information for A and B is complementary, and the information relevant to the waveform data (that D(+) and D(-) fire simultaneously) is distributed evenly and diluted but not skewed. Therefore the inefficiency factor of the two-photon absorption switch is like a tap. If the efficiency of the switch is less (determined by the energy level of the photon beam), then the stream of relevant information becomes less, but this does not cause an overall error in the results. Similar analysis can be done for the more complex combinations of path-structure.

Another way to think of this is that a classical perspective is selected by the observer. This is that the observer selects observation for the event that ports D(+) and D(-) fire simultaneously. The waveform displays the history of paths that contribute to this outcome. The reason this outcome

is possible is that the path-structure of the two concatenated interferometer structures is entirely operating at the level of quantum interference. There is no classical interference to collapse the overall system. Statistical data is collected on this using the technical of weak measurement.

It can be seen that the scenario, of mutual measurement by the particles themselves, goes far beyond the simple framework of an observer disturbing the system and causing it to collapse to a classical form. A complex interference pattern is created between the waveforms on two interacting quantum-level devices, and a particle description never applies to the paths except at the point of detection which is at the ports D(+) and D(-).

Filling in the Table of Probability When (D+) and (D-) Fire (see Table 1 below)
From the mathematical analysis by Aharonov, a probability table is created for the combination of where the particles were when D(+) and D(-) fire. The experiment by Lundeen and Steinberg is intended to confirm the results in the experiment using "weak measurement" techniques.

It should be noted that Table 1 describes the distribution of the waveform in statistical terms, and particles do not actually exist on any of the paths in the classical sense. The tables only represent what the waveform looks like as a numerical distribution pattern.

Table 1: Theoretic Weak Values from the Aharonov Paper
I = inner path, O = outer path

	N(I-)	N(O-)	
N(I+)	0	1	1
N(O+)	1	-1	0
	1	0	

The Aharonov paper refers to positrons and electrons. One interferometer passes electrons and the other passes positrons. For each interferometer, when its port (D) fires, we (classically) conclude that its particle must have been on an outer path and the other particle must have been on the inner overlapping arm (weakly measuring the particle-system of the particle on the outer path). Then there is an equal probability that the port (C) and (D) will fire for the particle that was on the outer path. We conclude the same for the opposite arrangement of inner and outer path-events.

However, the description in the previous paragraph is of a quantum event-structure from a classical perspective. This is the case since the table refers to the event structure when both (D+) and (D-) fire simultaneously and not (classically) the arrangements that apply when particles are either both destroyed (the value of 0)) or individually not destroyed (the separate values of 1). We have a table that describes the event structure from both a classical perspective (of real particles) and quantum perspective (of a waveform). The exception is the value for N(O+):N(O-)

of -1. There is no classical basis for a negative probability. The table has a simultaneous flaw that it reflects both a classical and quantum version of the events. The table is a collection of two descriptions that are not members of themselves and once again we find a not-set construction that supports the thesis of this paper.

Collectively for the observation that D(+) and D(-) fire simultaneously the first two conditions set out by Aharonov are met:
(1) • The electron is always in the overlapping arm.
(2) • The positron is always in the overlapping arm.
These are the single occupations of the two overlapping arms taken separately. For each particle, this must add to one for the appropriate cells on the bottom-left and top-right.

The inner cells list the joint occupations of the arms. For the inner upper left cell, which lists joint occupation of both inner arms, both particles are destroyed, and the value of this cell zero. Joint occupation of the outer arms results in the value minus one. This a purely quantum result since a classical value cannot be less than zero.

We see that the third condition stated by Aharonov violates the first two.
(3) • The electron and the positron are never both of them in the overlapping arms.

All of this adds up to indicate that what is happening is an interaction of paths within a complex quantum-level waveform. This is not a collapse scenario, and we should not think of the particles as actually being present on any of the internal paths. True to the framework of all not-set constructions, The criteria listed by Aharonov are individually counterfactual but refer to a structure in which they are held in common.

The only context in which collapse applies is that the observer selects the criterion that both D(+) and D(-) fire and then analyzes the quantum-level waveform that contributes to this outcome.

The Experiment by Lundeen and Steinberg

The technique of "weak measurement" used in the experiment by Lundeen and Steinberg is key to give access to quantum-level structure for the simultaneous firing of D(+) and D(-). Direct measurement would destroying this structure by raising it to the classical level. The paths are only weakly disturbed and the results statistically analyzed over many samples.

In documentation of the experiment, a significant point of validation that a quantum structure is being directly observed, without collapse, is the negative value close to (-1) found in the lower right box (when both particles are on the outer paths). Other values equally reflect a quantum-level action since the observable of the experiment is the simultaneous firing of D(+) and D(-). This observable is not possible in strictly classical terms.

Analysis Under the Geometric Model

In this writing, the claim is made that the only source of experimental error is the accuracy of the

interaction-free measurement (IFM) technique (this will be discussed). The inefficiency to the two-photon switch does not contribute to error. Rather this factor, in the data, points to a new principle in the basis of the quantum-level structure that can only be understood in the framework of the geometric model presented below.

The findings in the paper by Aharonov, for simultaneous firing of detectors D(+) and D(-), are summarized in Table 1. The results for the experiment by Lundeen and Steinberg are summarized in Table 2 . An interpretation of the results based on the geometric model in this writing (presented in a later section) is found in Table 3.

Application of the geometric model is critical to understanding what is going on between the pure theory and the experimental results in Hardy's paradox.

The geometric model is based on the unit circle that also contains an internal hexagonal configuration and an associated central null-state. The identification of this structure follows from analysis of the process of self-organization that is fundamental in the transformation from a quantum to classical state.

There is a natural force and trajectory that applies beginning from a null condition of a not-state that crosses dimensional boundaries and emerges to the classical level. The geometry is the end result of this process of self-organization (represented to the limit of this two-dimensional model). This concept is explained in a later section. In the interim, the final form that results from this process is simply applied to Hardy's paradox.

This structure represents another *footprint* in identification of the fundamental property of the native universe as an immaculately disordered state. A diagram of the pertinent parts of the geometric structure, with notations, is shown below.

What Differs Between the Theory and Experiment

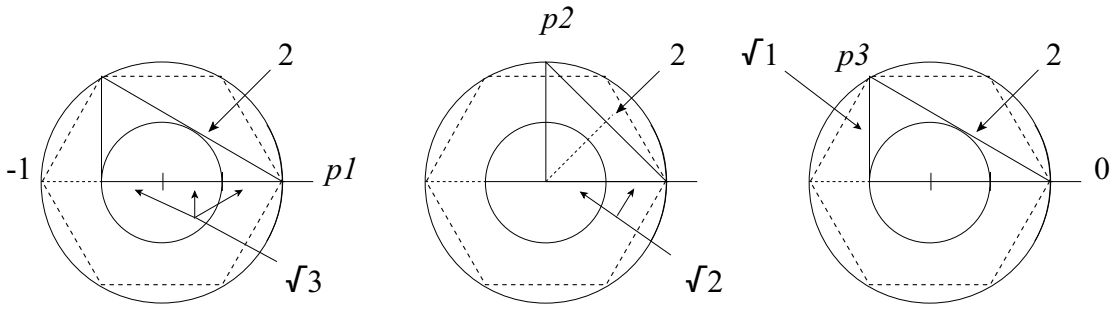
There is a difference between the theoretical calculation (the pure values) and the experimental results (observational values) which reflect the sub-classical geometry described below and the position of the observer between the two concatenated waveforms for Hardy's paradox. The quantum-level framework of the observer is of rotation in a non-distributive structure. Each position of observation has unique angular relationship to the event which is the simultaneous firing of D(+) and D(-) represented in the table of values.

The values in the boxes of the tables are non-distributive quantum-level uncertainties for the overall outcome chosen for observation. In a purely classical framework, this would not be the case. We would find a combination of events that had a distributive relationship across possibilities.

Complete uncertainty for an event does not infer that the event never happens. Rather, it infers that, in classical terms, we have no knowledge whether it happens or not. This is an equivalent

framework to Heisenberg's uncertainty principle in which, the uncertainty of position and momentum are complementary variables.

The equation for the relationship of the angular position of the observer (in rotation through the waveform) to the linear components of the table is calculated using the cosine law. The angle of the observer subtended to the event is $P(\theta) = \cos^2 \theta$. This yields the statistical uncertainty of the event to the observer expressed as the combination of the two sets of paths. When the angles created by the rotation are assessed within the geometry, they explain the discrepancy of the pure values in Aharonov's calculations to the experimental results.



the observer's position (*p*) as rotation through the simultaneous firing of both ports D

Both Out (not destroyed)	(One Out One In) (not destroyed)	Both In (same path and destroyed)
$(\sqrt{3})/2 = 0.87$	$(\sqrt{2})/2 = 0.71$	$(\sqrt{1})/2 = 0.50$
$(0.87)^2 = 0.75$	$(0.71)^2 = 0.50$	$(0.50)^2 = 0.25$
$= \cos(30)^2$	$= \cos(45)^2$	$= \cos(60)^2$

Figure 3. In Hardy's paradox the observer is uniquely placed between two parallel quantum states. True to the signature of quantum structures, the observer no longer has a consistent position-of-observation for the event-structure that D(+) and D(-) fire. Instead, the observer rotates. Each position represents a different observational contribution in this rotation and equivalently a different experimental result in the data obtained. The positions are *p1*, *p2* and *p3*. Of note, the trigonometric values assigned to the sides of right triangles are not magnitudes on a consistent and common number line in this representation. They are quantum-level vectors that either cross dimensional boundaries eccentrically (having √ assigned) or are neutral (not requiring the square root function). The value for "Both Out" of 0.75 takes a negative value in the results because the projection from the position of observation is to the negative side of the geometric structure. There is a geometric difference between the Argand plane and the geometric model. For the Argand plane the central null-state is hidden at (0,0i).

The values in the experimental data are at odds with the pure values because of the fundamental difference that quantum formalism and the geometric model. In brief summary, the difference is that quantum formalism is a system of fixed and preformed elements and operations; whereas, the geometric model is based on the process of development from a null state to emergence of the classical state. This development crosses dimensional boundaries that have a not-set or Russell set configuration that cannot be accurately represented mathematically.

The Tables and Their Geometric Interpretation

Table 1: Theoretic Weak Values from the Aharonov Paper

I = inner path, O = outer path

	N(I-)	N(O-)	
N(I+)	0	1	1
N(O+)	1	-1	0
	1	0	

Table 2: Weak Values from the Lundeen and Steinberg Experiment

(raw values to 3 decimals)

	N(I-)	N(O-)	
N(I+)	0.243	0.663	0.882
N(O+)	0.721	-0.758	0.087
	0.925	-0.039	

Table 3: Corrected Pure Values Based on the Geometric Model

	N(I-)	N(O-)	
N(I+)	$\cos(60)^2 = 0.25$	$\cos(45)^2 = 0.50$ $0.663 \times 0.721 = 0.478$	0.75
N(O+)	$\cos(45)^2 = 0.50$ $0.721 \times 0.663 = 0.478$	$-[\cos(30)^2] = -0.75$	-0.25
	0.75	-0.25	

Table 1 gives the theoretic values based on the paper by Aharonov. Table 2 gives the results of the experiment by Lundeen and Steinberg that demonstrated Hardy's paradox. Table 3 is a placement of the experimental results in the context of the geometric model found in this writing.

The difference between the theoretic values, in Table 1 and the experimental results, in Table 2 (which have been claimed to represent experimental errors) is explained by the calculations in Table 3 that are based on the geometric model. The agreement between the experimental results and the calculated results in Table 3 validates, very closely, the geometric interpretation. The pure values in Table 1, derived formally, do not account for the geometric relationship of the position of the observer to the event that is observed. The central null zone in the geometry forms an exclusion between the observer and the observed event that prevents direct observation. Table 3 is best thought of as a listing of the uncertainty that applies for knowledge (by geometric position) to the specific event listed in each box.

Each value is a numerical expression of the angle subtended for the observer in rotation around all the positions that apply. The values are better referred to as "levels of uncertainty" than probabilities for the event.

[N(I+) and N(I-)]

The theoretic value for [N(I+) and N(I-)] (that both particles are on the inner paths) is zero. In this case, we would have no uncertainty whether or not the particles are destroyed. Going back to Aharonov's three counterfactual statements on the complete description of what occurs (Note: decisions are always classically based.):

A) If we decide that the particles should be destroyed then we are certain this is the case (the frame of reference that both particles were on the inner paths), and

B) If we decide they were not destroyed (since D(+) and D(-) do survive) then we are also certain this is the case (but again, in only one of two frames of reference for context - that both particles were on the outer paths). Nevertheless, a context for such conclusion must be chosen, and there are two paradoxical perspectives to choose from. Remembering that information always has a classical basis, and the table is of quantum-level structure, in the final analysis, when both outcomes are taken together we have no information at all.

As can be seen in the geometric interpretation, the level of uncertainty is higher than given by the pure value of zero in Table 1. In other words, there is some uncertainty to outcome whatever we think this outcome is. This is born out by the experimental values in Table 2 and the recalculated values in Table 3.

[N(O+) and N(O-)]

The theoretic value for [N(O+) and N(O-)] (that both particles are on outer paths) is minus one. In this case, we would have complete uncertainty whether or not the particles contribute to the firing of D(+) and D(-).

A) If we decide that this combination of paths contributes, then we would have no certainty for that decision.

B) If we decide that the combination does not contribute then similarly we would have no certainty for that decision. Again, the pure value of Table 1, does not reflected the experimental value in Table 2 or the recalculated value in Table 3. There is a lower level of uncertainty (some certainty exists) relative to the pure value.

It deserves repeating that, at a deep level within the sub-classical geometry, the readings are positions of observation as the observer rotates through the total of possibilities for uncertainty. This uncertainty combines the paradoxical conclusions that the particles were destroyed when it is known that they were not destroyed (since the tables are based on the condition that ports D(+) and D(-) did fire simultaneously).

The cosine rule: The rule for the value of uncertainty for the event is based on the angle subtended by the observer to the event. For probability (P), $P(\theta) = \cos^2 \theta$.

For rotations $\theta = 30$ and $\theta = 60$

The *values for uncertainty* are created by the degree of angularity attached to the point of projection from the observer to the event. The values found experimentally and those calculated for the angles of 30 and 60 degrees match exactly for the down-diagonal. The minus value is applied to $[\cos(30)^2]$ in order to conform with the geometry. These results do not conform to the pure values predicted under quantum formalism. The discrepancy owes to the fact that the observer is uniquely placed within the event-space which includes the exclusion of the central null zone. Quantum formalism does not account for this geometric structure. The observer is placed within the structure in the sense that it is only weakly disturbed which allows the collection of quantum-level data.

The level of quantum-mechanical uncertainty for *p3* in the geometric values of Table 3 is 0.25 Here both particles are on inner paths and, classically, should destroy each other. The geometric value for the level of uncertainty for *p1* is 0.75. As this value is generated on the minus side of *x* axis, in the model, it seems reasonable to assign a minus sign. Both values are very close to the experimental values.

In purely geometric terms, the position of the observer as either eccentrically or concentrically placed for the horizontal axis is an important factor in the calculation of the level of uncertainty. It is a measure of the "sliding scale" of uncertainty associated with the positions. For rotations 30 and 60 degrees, the position of the observer is eccentric. Across these angles, the more removed the observer's position from centre, the more uncertain is the recorded state.

The event-structure is uniquely quantum, with the additional factor that a level of uncertainty still exists (because of the exclusion zone). Neither system of logic (quantum nor classical) fully describes the result.

For rotation $\theta = 45$

For the rotation of the observer that yields angularity of 45 degrees at *p2*, the observer has a concentric position for the horizontal axis (albeit this axis is obscured by the intervening null zone below *p2*. Accordingly, there is no distinction between the two values on the up-diagonal, and it seems appropriate that the two values can be combined as a product of two (0.721x0.663) which yields the value 0.478 and is very close to the value in Table 3.

Of note, a similar structure of simultaneity for the up-diagonal is found in the simpler half-silvered mirror experiment. When there is no interference by the observer, the path of the photon is symmetrical across two listings of uncertainty. Of course, in the half-silvered mirror experiment there is no mechanism that allows "weak measurement" across the path-structure. The two paths are summed for reading their effect at the classical level, as (-|F>) + (-|F>) = -2|F>. The two halves of the path-structure contribute an equal portion of uncertainty to the outcome. What is linearly additive in that format, is multiplicative in Table 3. In other words, what is linearly additive in quantum formalism becomes multiplicative at the classical level.

Rotation and Uncertainty

Rotation of the observer in Hardy's paradox is equivalent to the rotation of the calcite crystal as discussed for Bell's inequality. Uncertainty for the value of polarization develops as a crystal is rotated from zero degrees. In the case of Hardy's paradox the rotation of the observer results in increasing uncertainty to what has happened to contribute to the simultaneous firing of D(-) and D(+).

The IFM factor - for the diagonal-up boxes

The IFM error rates of 95% ± 3% for [N(O+) and N(I-)], and 94% ± 4% for [N(I+) and N(O-)] for the experimental values, in Table 2, are within the limits for the pure geometric values in Table 3 as follows:

Note: the justification for squaring the experimental values is based on the fact that the raw values must be raised to the classical level. This is equivalent to the similar operation of squaring the modulus when calculating a classical probability for a superposition of paths at the quantum level.

1. The [N(O+) and N(I-)] ("lower" up-diagonal box) probability is recorded as 0.721 and is corrected (by squaring) to 0.519. This represents an error rate of 3.8% for the value predicted geometrically (in Table 3) using the cosine law. If we use the interpretation of this writing (the alternative calculation of 0.721x0.663 = 0.478), then the error rate is 4.4%, and still within limits established by Lundeen and Steinberg.

2. The [N(I+) and N(O-)] ("upper" up-diagonal box) probability is recorded as 0.663 and corrected (by squaring) to 0.439.

This is an error rate of 12.2%. The stated lower range for error (94-4)% is 10%. This is only slightly outside the pure value (using the cosine law) that is established in the geometric model. Again, if we use the interpretation of this writing for which the alternative calculation 0.663x0.721 yields 0.478, then the error rate is 4.4%, which is well within limit of the geometric value.

Bell's Inequality and This Experiment

In Hardy's paradox, the observer has rotational locations within the waveform of two concatenated quantum-level event-structures. The central null-state is responsible for the

discrepancy between the pure values in Table 1, and the experimental and geometric values through the three rotated positions of the geometry. In other words the quantum description fails, in its frame of reference, to account for the presence of the classical observer. This is the opposite situation to the paradox found in Bell's inequality. In that case, the classical description appears to fail, in its frame of reference, because the experimental data does not account for a quantum-level action.

The apparent failures in each case are not formally resolvable through discovery of any new principle. Rather they point the presence of dualism across which the mechanism of paradox prohibits resolution.

Hardy's Paradox in the Framework of all EPR Structure

The mechanism at the centre in all EPR structures

All EPR structures have at their centre a structure that is closed to observation of the interior. It is closed in the sense that no discrete knowledge (in any appropriate form) on the elements within is allowed. The mechanism for this prohibition is that a paradoxical framework applies between the quantum-level and classical-level frameworks. The absolute closure of the interior of the quantum-level structure at its dimensional boundary to the outside classical level means that the boundary can be referred to as an "infinity" for the quantum state. All information is prohibited.

In EPR experiments, other than Hardy's paradox, the observer has an operational choice of either:
1) not disturbing the system and thus leaving it closed to outside influence and knowledge or,
2) opening the structure through some act of disturbance including observation. The consequent is that the inside system of order is destroyed. It is dimensionally raised to the classical level, which also introduces the ability of the observer to have "knowledge".

For purely classical structures, the mechanism of paradox (the mechanism for the exclusion to knowledge) equally introduces a unique signature of "boundary". In this case the boundary is also an "infinity". However, where the boundary could be clearly identified in the case of quantum structure, it takes a "non-settling" format, in the classical setting. It is displayed in two ways:
1) In static terms, it has the form of alternatives in a cycle of time (for example when there are two alternatives such as the flipping of a coin) and,
2) in dynamic terms, it has the form of non-containable growth or contraction (for example, in space and time). For mathematical representation this growth requires the application of a power function. Of note: An example of contraction is gravitational acceleration, and an example of growth is the flow of time.

The twist in Hardy's paradox

Hardy's paradox presents a new twist to the framework of EPR structure. Because the information obtained from observation is internal for the quantum state, the various positions of observation within the state are non-distributive. This contrasts to the complementary classical (distributive) format in which locations are distributive.

Conclusion

The experiment conducted by Lundeen and Steinberg was intended to verify the paper done by Aharonov on Hardy's paradox. The calculations in this paper are know to be correct under quantum formalism, and although the experiment contains elements that are supportive of the theory, there remains a significant divergence between the theoretic and experimental results. In this writing it is claimed that the results very accurately point to a new principle that can only be understood through examination of the role of paradox in the quantum-level geometry described above.

Each position of observation within the structure is quantum-based. This means that there is no consistent translation across the state in classical terms. Consequently we see paradox is openly displayed for the relationship of paths-taken (the history of paths) to the event that is assessed which is the simultaneous firing of D(+) and D(-).

In the most general terms, the firing of detectors by particles is simply a measure of the symmetry of uncertainty that applies across the two concatenated devices. This symmetry which includes eccentric and concentric locations for the observer must be fully accounted for in the experimental probabilities. For this experiment the "reality" observed, at the classical level, is the summation of all the degrees of uncertainty that apply when D(+) and D(-) fire simultaneous.

Hardy's paradox is another *footprint* to the fundamental nature of the universe and reveals the role of the hexa-orthogonal structure that spontaneously arises out of it. The hexa-orthogonal structure is the most general framework of organization for what we observe in Nature. Its structure is the hidden initiator of all the far more complex systems that follow at the classical level.

The opportunity for validation

The experiment by Lundeen and Steinberg presents a unique opportunity to indirectly valid the model of this writing. If the efficiency/inefficiency ratio of the two-photon switch can be arbitrarily adjusted, and the same results are obtained, then the switch is not a factor of experimental error and we must look to some other explanation for the discrepancy between the theoretic values and the experimental results. This explanation is as described above.

Other EPR experiments

There are numerous other EPR experiments in which the same logic can be applied to demonstrate the mechanism of paradox at work. In each case a dimensional boundary applies between the quantum and classical bases of a common state.

Infinity

The common definition of infinity is inaccurate and incomplete. A common definition, as found in Wikipedia, is that infinity refers to, "something without any limit". The definition of infinity should not rely on comparisons of "size". Statements such as "bigger than anything we can count" or giving the endless examples of such incompleteness do not lead to a deeper

understanding.

A much better definition that references the role of paradox is as follows:

An infinity is the identity of any system in which complete reference to property is categorically prohibited from enumeration.

The focus of this new definition is that a "prohibition" prevents any form of statement and observation that is comprehensive. As has been discussed above, the self-circularity of such prohibition applies in many different contexts. At the grand scale, it applies to finding a "theory of everything". As well, it can be studied in the very limited context of well-defined mathematical arguments. The same framework of prohibition applies. In each case a boundary exists based on a paradoxical relationship of elements. In the simplest case, for dualism, two elements refer to the same state but are not common members of it.

1. The Möbius Strip as an Infinity

The Möbius strip is created by twisting the ends of a strip of paper 180 degrees and joining them. It is an excellent "limited" format to demonstrate how we should think of the meaning of "infinity", that it is not based on size. Rather, infinity is a format for closure of a structure or space such that the containment or listing of all its parts is not distinctly identifiable even though the structure of reference is identified as complete.

For the Möbius strip, two sides are discrete and readily identifiable as long as no attempt is made to trace path within (on) the structure. However, if the observer takes position within the structure at any location an attempts to trace path, only one side is observable. From a position outside of the structure, it contains two sides. From a position inside, it contains only one. Two frames of reference (inside and outside) for closure of the state apply that are paradoxical to each other. The mechanism for creating this paradoxical construction, as with all such examples, is that a flipping action has been applied to the elements of the state, which in this case is to join the ends of the strip at 180 degrees from normal.

The Möbius strip, as an infinity, extends with a limit (by tracing path back to origin). However, this limit hides a larger framework of "sidedness" that is not observable from within. The Möbius strip is a Russell set construction because two parts are included as distinct members of the structure, but we cannot discern the difference (have no knowledge) of these parts (from within). They are closed to observation, and the two sides are not members of themselves (to the observer) for the tracing of path. The Möbius strip is a very small *footprint* of the universe from which it takes its form.

2. Cantor's Diagonal Slash Argument as an Infinity:

Cantor's diagonal slash argument defines an infinity in its vertical listing. This is the listing of the natural numbers, and in theoretic terms it is properly constructed. Each row in the listing is then associated with a real number between zero and one. The diagonal across the rows and down the

column is then selected and modified according to an arbitrary rule that categorically flips the property of all the elements in the number.

For example: All 1s are made not 1s (they become 2s), and all not 1s (2s or otherwise) are made 1s. Thus the general rule of operation is that the number is transformed based on its identity as a simple dualism of parts (1s and not 1s). These two parts are each categorically flipped from is to isn't. This new number is then not a member of itself, and analysis of the vertical listing shows that it is also not a member of the set within which it is a member.

Illustration

For the diagonal number 0.192 . . .

The diagonal number is necessarily a member of the vertical listing since this listing is known to be infinite. This diagonal number is used in the second part of the argument to construct a new number using a simple technique that transforms the property of each of the individual digits of the diagonal number and thus of the number itself, into a new number.

Natural position	real value
0	**0**.0 0 0 0 . . .
1	0.**1** 8 4 5 9 2 0 3 4 8 3 9 4 . . .
2	0.3 **9** 5 7 8 2 0 9 4 7 3 8 4 . . .
3	0.5 6 **2** 3 8 2 1 5 2 4 6 7 8 . . .
4	0.2 7 3 **4** 9 0 1 2 6 8 4 3 7 . . .
5	etc.

The switch from is to isn't for the diagonal number 0.1924 . . . results in 0.2111 . . .
This number is not a member of the original infinite set represented by the vertical column which is shown by trying to place the newly created diagonal number in the vertical listing. The new number on the diagonal should be the same as some number (not shown) in the vertical column since the column is an infinite listing. However, this is not the case because, for the diagonal position (shown in red) in any number listed in the vertical column, the property of the digit is reversed to the property of the same digit in the newly created number.

Regardless of how far down the list this comparison is made, in any horizontal number the same situation will be found for the digit at the point of the diagonal. Thus categorically, the new number cannot be a member of the vertically listed, infinite set. Paradox arises because a set that should be an infinite listing does not include all of its members.

Cantor's intention was to show the disparity between a structure represented as an infinity and our ability to rationally construct it so that it is complete. This is a form of the Russell set in the following way. The listing of real numbers is complete because it is an infinity as identified by the column of natural numbers. However, the listing of real numbers is found to be incomplete. Therefore, the real-number listing that we see in the real-number column is not a member of

itself as the infinity it is supposed to be. It is then not to be included in itself as a set. However, it is an infinity as specified by the column of natural numbers and should be included in its specification as a set. Thus, there are two placements for the set identified in the real-number column, and we cannot decide which is correct.

3. ($\sqrt{-1}$) as an Infinity

How does ($\sqrt{-1}$) represent an infinity? It is an infinity in the sense that there is an absolute prohibition to information on its properties. The internal structure of ($\sqrt{-1}$) is paradoxical. Infinity, defined in this writing, has two equal frames of reference. Both take there reference from the fact that absolute and singular "information" on the totality of the state is prohibited.

In the first, we are prohibited from creating an infinite listing that includes all of the parts of the state. A boundary "horizon" exists (for "information") beyond which some members will always be excluded regardless of how comprehensive our framework is or how far the listing extends. In the second, there is an absolute prohibition to information on the internal structure of the state. The observer is also prohibited from "information" by a boundary. In physical systems and their associated mathematical representation, this boundary is dimensional.

The property of ($\sqrt{-1}$) falls into this second category of prohibition on information. The interior construction of the function is categorically closed to our assessment of its properties. The only way we are permitted to rationally represent the term (as classical) is to arbitrarily collapse it. The identity becomes -1. The consequence for raising it to the classical level is that its internal structure, at the quantum level, has been destroyed. There is no rational bridge (transformation) across these two frameworks.

The Development of Complexity

Complexity develops from a null-state, called the native universe, which is without form. It is immaculately disordered which means that nothing we associate with the structures of space exists. There is no angularity and no pre-existent system of laws for the development of order. This does not mean that order cannot develop. A process of self-organization that has a force and trajectory can be identified. In this section the pattern for this self-development is discussed.

The process of development is sequential across dimensional boundaries that are observationally hidden to each other. The limit within each boundary is an infinitude, and an evolution occurs that results in the accumulation of potential. In subsequent spaces, across such boundaries, complexity builds based on the potential that has been is previously subsumed into the overall structure through self-organization. The relationship of such spaces is as identified in Russell's paradox that the spaces are not members of themselves but refer to the same overall structure in this process of development.

The sequential development of the native universe is a process of least or minimal action - not a chaotic process. Chaotic processes rely on the existence of a classical-like background of complexity in which the event-structure is expressed randomly within the total collection of

possibilities. By contrast this is a process of building on quantum-like fluctuations.

Angularity and Spin

The condition of paradox for the native universe (as a null-state) induces both a force and outward trajectory. The first stage of this trajectory is the development of angularity. As stated above, this is not a random process, A smooth curve develops incrementally in a process of quantum-like least action. The angularity that develops does not pre-existence. As angularity is initiated and begins to build, angularity that already exists, from this building, forms a minimum structure. Accordingly, new angularity must be distinct for its direction in order that it does not lend structure that has form above quantum fluctuation. Consequently, angular develops as a smooth curve (of new direction) in the space.

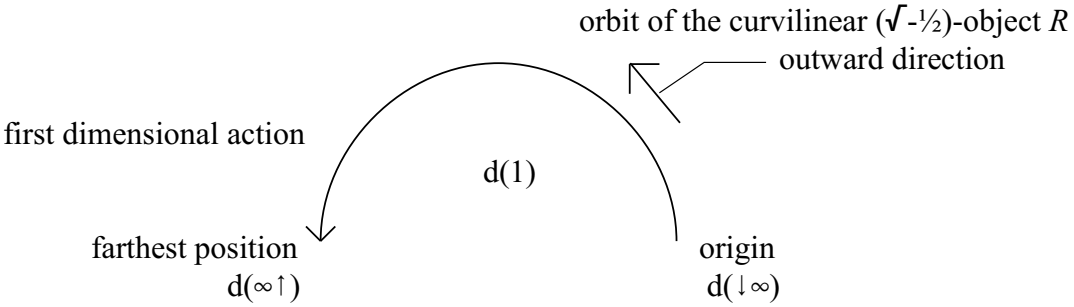


Figure 4. The first plane of dimensional development from an absolute null-state is illustrated. The angular space is contained by the indicated lower and upper limits, which are true boundaries to infinities.

The first curve culminates as a semi-circumference. At this point a dimensional boundary has been reached. The development of any angularity beyond this point is a return to origin, and as such, is not new angularity constituting outward development. Consequently the first half of the curve is dimensionally closed to the second. This is a dimensional bifurcation allows a second semi-circumference to develop without contradiction to the framework of the first for outward trajectory.

The second (√-½)-orbit is a mirror image of the first. Because, the first one-half orbit is closed ("collapsed"), across an infinity to the second, the first curve represents a dimensionless point interpreted as a (√-½)-spin from the dimensional level of the second (√-½)-orbit.

next page

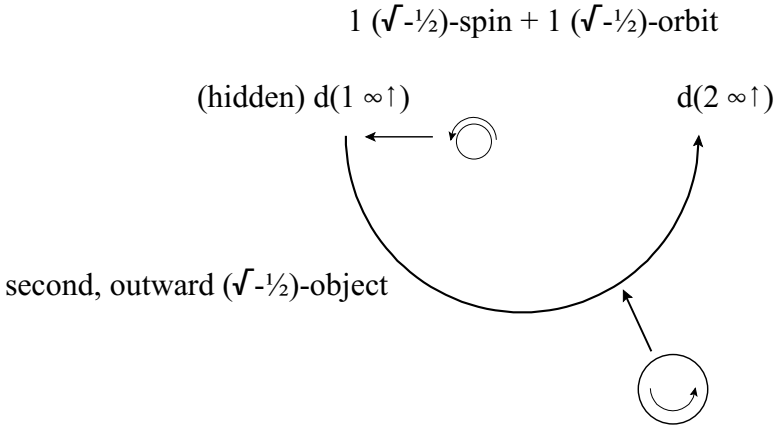


Figure 5. Development of the second √-½)-orbit object is illustrated. The entire summation of this and its previous hidden half has the symbol d(3) at its upper infinity (see Figure 6). This summation is the original native null-state displayed as an semi-circumference. Two (√-½)-objects have accumulated complexity as a sub-classical cycle.

The first dimensionally lower (√-½)-spin object has been carried forward on the second curve as it develops in return-to-origin. This culminates at the origin. A space, which contains the original null-state, has developed two levels of dimension each with its own signature structure. The first space, [d(↓∞)→ d(∞↑)], is collapsed in respect to the open structure of the second space. In Figure 5.

Translation

The next structure that results from outward trajectory and quantum-like fluctuation is a second semi-circumference. The mechanism is "translation". This is represented, in Figure 6, as the shell **d(3)**.

Linear Projection

Once the trajectory has returned to origin (Figure 5), all angularity has been developed for the two-dimensional plane, and this framework of development is dimensionally closed at an upper infinity. This is the structure represented statically as the (**d3**) circumference or shell in Figure 6. Across this new boundary for an infinitude, the framework shifts from angular to linear trajectory. This is in keeping with the principle that outward development represents a non-chaotic quantum-like fluctuation of progressing self-organization. The principle of least action that now applies is a tangent forms from the closure of the two semi-circumferences.
Note: The previous structures, as those which follow, are observationally closed to each other.

next page

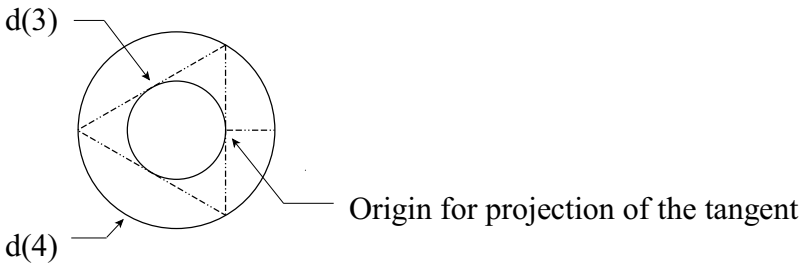


Figure 6. The shell (**d3**), forms an enclosure that contains the original null-state as an open space of "no form". From this state the new "minimal" structure that develops has trajectory first as "tangent", and then as reflection of the tangent.

Reflection

Linear trajectory of the tangent culminates at the limit of the enclosure that forms the outer circumference. At this point the property of reflection becomes the new basis of the trajectory. Angularity subtended at 60 degrees is the minimized "least action" for the difference in angle between vectors that are developing. These are identified in Figure 6. For the resulting structure, tangents have been formed that project from the inner circumference to the outer circumference. This outer shell is a new boundary of dimensionality to the overall accumulated development of the space.

Rotation

The next phase of non-chaotic, least-action development is rotation of the first "tri-vector" reflection to form a second tri-vector. This produces the hexa-space identified in Figure 7.

Closure to a Classical Point

The structure in Fig. 7 is a sub-classical "canopy". As well as the tangential projections from the inner to the outer shells, it also includes the projections around the outer shell between the points of the tangents, and projections directly across the structure to the inner null-state. This inner null-state blocks the direct projection of a vector from one side of the outer shell to the other side. This two-dimensional structure is the inner, multi-dimensional, sub-classical and hidden support for what would appear as a single point at the classical level. The point so produced at the classical level, includes:

1) angularity and spin (forming orbit then spin)
2) translation (as a copy of the semi-circumference)
3) linear projection (creating tangent)
4) reflection (of the tangent around the shell) and,
5) rotation (creating the outer circumference).

Finally, this is all closed as the potential for the emergence of classical space. The space represented in Figure 7 is dimensionally collapsed in classical terms to an observer. It is also the basis for the potential displayed as dynamic action on the classical plane. All structure on the classical plane relies on this hidden framework of spontaneous self-organization.

The key to understanding the fundamental construction of sub-classical (quantum) phenomena is the geometry behind it as shown in Fig. 7. Within this geometry, and at its centre, is found the expanded space of the immaculately disordered "native universe". This region is absolutely inaccessible and collapsed as viewed from the outside and, as such, has no reference to the outside. It is the boundary of an infinity that has no inner form.

The Geometry and Mathematics of Sub-classical Space

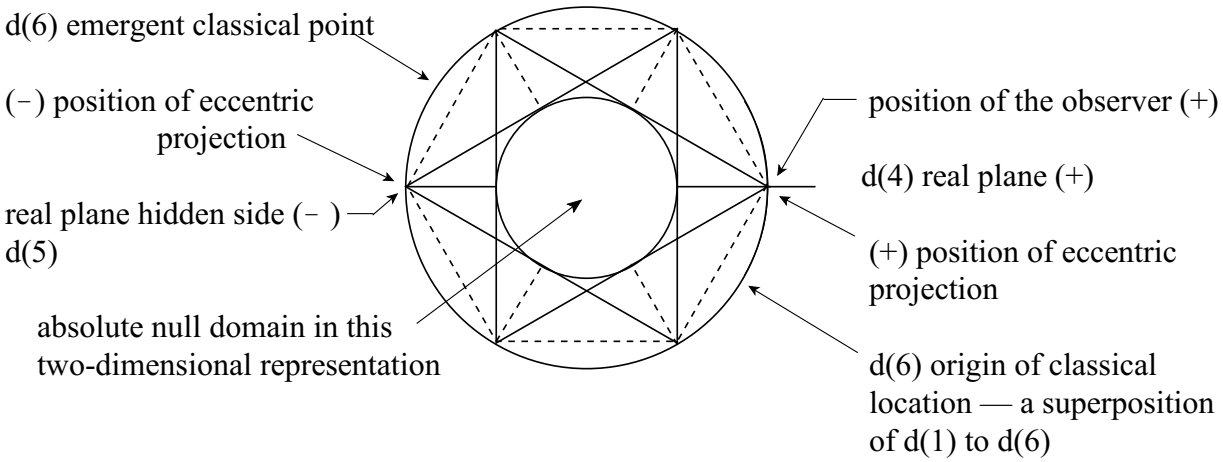


Figure 7. The interior structure required to support the canopy or shell of a classical point is illustrated. The internal structure is sub-classical. In 360°, there are six locations on the circumference. The space is "hexa-orthogonal". This structure is displayed in a classical format, i.e. all the contributing dimensional levels are contained on a fixed, composite plane.

The entire shell out to d(6), which is hidden and collapsed within the point (0, 0), of the classically based unit circle. It is the precursor, and hidden basis on which classical space can emerge along with its own framework of orthogonality. The delineation of the structure within this dimensionless "point-state" makes it possible to understand the mechanism of creation behind a variety of structures. These include the basis of hexa-orthogonality in the structure of *pi*, the rotational nature of phenomena found in EPR experiments, and the force of gravity as a phenomenon that replicates the outward trajectory of self-organization identified in the structure. These features will now be discussed.

The sub-classical structure of *pi*

The constant *pi* (π) is the irrational number with the approximate value (to seven decimal places) of 3.1415993. It is the fundamental factor of equivalence for the transformation between rotational and linear structures.

The unusual hexa-orthogonal structure identified in Figure 7 "opens" the hidden relationship of closed locations and the vectors between them which establishes the identity for *pi*.

Circumference is the geometrically simplest framework for the identity of any structure that is "closed" (meaning that it does not extend in space). In similar context a "point" location represents non-extension in space but without interior. The outer circumference of Figure 7 is the canopy that defines what does not extend.

The equivalence or transformation for what is closed and does not extend to what is open and extending, of same structure, must account for the basis of locations and vectors that contribute. It is a ratio of how many vectors contribute to closed locations in the equality between rotation extension. Six sub-classical points on the outer circumference each reflect to produce 3 vectors around the structure of the outer circumference, as equilateral triangles. There is also a single horizontal plane having two parts across the null-state, and there are two vertical axes (one on each side of the null-state). Note: the *iy* axis is not shown as it represents a different geometric basis in the sub-classical architecture. There is also opened null structure at the centre called the null-state. The sum of the points and vectors is shown in Illustration 1.

The origin of the value of *pi*, (to the limit of accuracy for the two dimensions of the model) is placed within the hexa-orthogonal geometry as follows:

6 (sub-classical points)	× 3 (projected vectors)	= 18 directions
+	+	
1 (null-state)	× 4 (projected directions)	= 4 directions
7 (total objects)		22 (total directions)

$22/7 = 3.14... \approx \pi$

Circumference/π = Diameter

Illustration 1. The sum of the points and vectors in the sub-classical space.

Thus, there are 22 vectors distributed among 7 points that create the canopy which is fundamentally equivalent to extension. This the fundamental constant for the relationship between rotational and linear projection.

The accuracy of this two-dimensional representation of π

The value of π established (22/7) is:	3.1428571...
The value of π calculated to 7 decimal places is:	3.1415926...
Difference =	0.0012645

As with all representations of π, the value established is only as good as the refinement of the number of terms that have been accounted for which is the number of decimal points calculated. In similar context, the accuracy of the model, in this writing, is dependant on the dimensionality of the sub-classical space represented or accounted for which is the two-dimensional plane. The locations and vectors between them that contribute beyond the limit of the two dimensional plane are relatively small corrections that would appear if the internal structures within the six points on the circumference were opened in a dimensionally more complex model.

Euler's Infinite Series for $\pi^2/6$

In 1740, Leonard Euler discovered the infinite series for *pi* shown below, and it has direct application in the geometric model of the native universe in this writing.

$$\pi^2/6 = 0 + 1 + 1/4 + 1/9 + 1/16 + 1/25 + ... = 1.644934... \quad (1.7)$$

cycle null 1 2 3 4 5 ...

There are two frameworks for expression of the native universe. One side of this framework is the "static" display of paradoxical relationship between elements that are members of a state but are not members of themselves. Any structure conceived as singular and universal, will internally always spontaneously breakdown into two elements that are not members of themselves. Each limited element in the universe must mirror the native form of the universe. The overall property of the universe is defined in each of its parts.

The other side of this framework of mirroring is non-static outward growth of complexity. This is complexity that does not settle down. As explained in a previous section, this outward growth is self-organizing which means that it does not rely on the pre-existence of any laws. This outward process has a sub-classical signature that begins as a formless, immaculately disordered null-state, and all form develops out of it. The process is the outward growth of trajectory that creates the forms of orbit, spin, translation, linear projection, reflection and rotation. The dimensionality of this state, seen as a canopy, closes as a sub-classical point-structure to the classical observer. At this stage classical complexity can emerge. Classical structures, in their own frame of reference, depend on the form and mechanisms of this sub-classical structure.

Displacement by the Force of Gravity

As for any limited static display of the overall property of the universe, the outward growth of complexity must also display to overall property of the universe. In other words, the classically-hidden process for the development of sub-classical form illustrated in Figures 4 through 7 will be found as the basis of dynamic action at the classical level. The relationship of Euler's series (illustrated in equality 1.7, above) to gravitation acceleration fits this profile under the geometric model.

From: http://en.wikipedia.org/wiki/Gravitation:
"Under an assumption of constant gravity, Newton's law of universal gravitation simplifies to F = mg, where m is the mass of the body and g is a constant vector with an average magnitude of 9.81 m/s2. The acceleration due to gravity is equal to this g. An initially stationary object which is allowed to fall freely under gravity drops a distance which is proportional to the square of the elapsed time."

next page

For a given unit of cycle of time, the outgrowth or accumulation of displacement is as follows:

(a) the growth of values for cycles:

n^2	=	null	1	4	9	16	25	...
cycle n	=	0	1	2	3	4	5	...

======

(b) contribution of the parts for growth to the value for cycle:

n^2	=	*		** | **		*** | *** | ***		**** | **** | **** | ****		...
cycle n	=	1	‖	2	‖	3	‖	4		...

Illustration 2. Part (a) gives the values for growth in displacement under gravitational force over units of time. Part (b) illustrates the format of this growth. Each of the asterisks represents one unit of displacement.

In part (b) of Illustration 2 we see the manner in which the process of growth has feedback on what was initially a singular element of the series. For each cycle, both the single element and membership within the single element are responsible for the values of displacement that develop. Thus, the single element is contained and replicates both as a set and as a member of the set. The number of sets displays the same property as the number of elements in each set. Thus, the elements and the sets are members of themselves.

For Euler's series (see equality 1.7), the process of displacement from and initial position is represented in reciprocal format. In this case we are following the framework in which the singular unit of displacement represents a fraction of the totality that develops in subsequent cycles.

Why *pi* squared and divided by 6?

It is claimed that the growth of complexity at the classical level is controlled by the sub-classical geometric structure in Figure 7. A dynamic process of the outgrowth of complexity occurs in a classical environment that takes its form from the sub-classical structure of the native universe. In answer to the question posed above, this dynamic process is one in which a sub-classical structure has been transformed for observation across a dimensional boundary. This is the same process of collapse found in EPR structures when the observer, by direct action, interferes within the quantum state. In that case the "squared modulus rule" is applied. In this case we square the value for *pi*.

The fact that the denominator is divided by six, points to the hexa-orthogonal basis identified in Figure 7 of the geometric model. The element that is under dynamic growth (as an object under gravitational acceleration) is one point on the circumference of the outer shell of Figure 7.

The geometric figure as applied to the process of simple gravitational acceleration demonstrates how sub-classical processes are manifested at the classical level. This is another *footprint* that points to the hidden form of the native universe.

Displacement as "Time"

In part (b) of Illustration 2 we see that the single element is contained and replicates both as a set and as a member of the set. The natural flip side of the rule for growth as gravitational displacement (when the elements and the sets are members of themselves) is that the elements and the sets are not members of themselves.

In this case the property of replication for a single element is not shared with the property of replication for the sets in which it is a member even though the elements and the sets are members of themselves. It turns out that this is fits the profile for the outward growth of *time* (see Illustration 3). The initial state, identified as cycle 0, contains one element. This element is the current "time". Thus, there is always a current time.

$$(1/2^n) = 1 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 \ldots = 2.0 \quad (1.8)$$

cycle 0 1 2 3 4 5...

(a) the growth of values for cycles:

2^n =	1	2	4	8	16	32	...
cycle n =	0	1	2	3	4	5	...

======

(b) contribution of the parts for growth of value for cycle:

2^n =	*	* | *	** | **	**** | ****	[********] x 2	...
cycle n =	0	1 ‖	2 ‖	3 ‖	4	...

Illustration 3. The series of accumulation when the element is not a member of itself is illustrated. The process is schematically represented in (b). The vertical bar (|) indicates the division across the number of sets, and within each set a doubling action occurs The collection of sets (which is always a maximum of 2 for any given cycle), does not share the property of the elements for replication within each of the two sets. This is contrasted to the growth in gravitational action.

Because the initial element is not a member of the set and the set is not a member of the element for the property of duplication, the observer's position in the overall process of inflation is anchored in one position. The result is that the inflationary growth of time is not observable. It is imaginary to the observer, and is not included in the frame of reference that establishes the process of growth. This is contrasted to the situation with the growth of displacement in gravitational action. In that case, we see the growth that occurs over successive cycles produces accumulation that is "real" to the observer, not imaginary as in the development of "time".

The growth of time is not directly displayed in the geometry of the hexa-orthogonal space found in Figure 1, but it is represented in the fact that a second ortho structure (a second unit circle) exists that is imaginary. Since the vertical axis of the structure in Figure 1 has only one real axis (in the horizontal direction), we can speculate that the second space has its real axis rotated at 90

degrees. As these two structures bifurcate, they are imaginary to each other. Thus, the growth of time has a larger framework than singularly represented in Figure 7.

The Dynamic and Static Structures of the Native Universe

The growth for space and time has been discussed, above, for the framework of this development that is contained within the boundary of the universe. The second side to this is the relationship of the universe to what is not contained within its boundary. This will also have both dynamic and static expressions.

Dynamic inflation

The boundary of the universe must expand outwards from its current state to satisfy the requirement, identified above, that it does not settle down. This is an inflationary universe.

A static relationship of two parts - not members of themselves

As well, the native universe should display a static basis for its form in which two parts are conjoined that are not members of themselves. This will be a universe that contains dark matter that is not a property "for membership" in the space of the universe.

The Growth of Space in Time

In the above argument, two series based on the summation of $1/n^2$ and $1/2^n$ for cycle (n) have been juxtaposed as complementary and paradoxical formats for the growth of displacement of the native universe. For space, this is specifically the growth of displacement when the elements defining this growth are "members of themselves". For time this is an outward growth of displacement when the elements defining this growth are not "members of themselves".

What remains to identify is how these two processes for "non-settling" are common members of the native universe. Their relationship falls within the framework of this writing. Specifically, they are not members of themselves since their individual identities for replication are not the same; yet they are members of the same universe as the two the two fundamental processes for outward growth.

We can surmise the effect of this relationship by examining the limits of the two processes. The limit (infinite value) for the summation linked to the growth of space is (1.644934...) and the limit for the growth of time is (2.0). For members in each format, the limit for accumulation is different. We can speculate that at some value for common cycle n for the two processes, accumulation as displacement in real terms for the observer (through gravitational acceleration) can no longer accumulate as imaginary for the observer (for time) even though a common cycle continues.

In this situation, time continues to accumulate classically within the universe of the observer - the observer continues to experiences in time. However, growth as accumulation of displacement cannot continue as real in the space of the observer. It has reach an absolute limit for accumulation in the common cycle. The future accumulation of space becomes imaginary. This

simple two-dimensional structure schematically illustrates the basis of singularities (black holes) in space-time

The difference between all of the correlated structures identified above is that paradoxical formats of structure have common membership in a single domain.

Emergent self-organization

Self-organization is the term describing the spontaneous development of order within an initially disordered state. The only requirement is that some fundamental law of organization applies between the nodes or elements of the state.

The "law" functions by creating a ordered dualism of fundamental parts in which one part is displayed as a singular state and the other part is "hidden" from the display as the precursor. This format is also mirrored in all the structures discussed this far, and it is a fundamental feature of the universe itself. The square root function is the mathematical expression of the "hiding action" is. Of significance, the square root function is also the fundamental basis of the transformation responsible for all quantum-mechanical structures. In that case, the function, the square root of minus one (i), is attached. The same general principle of hiding the parts of a state applies. In the case of quantum states these parts (paths or the observables of some property) are referred to as "entangled".

The Kauffman experiment

Stuart Kauffman describes an experiment that demonstrates self-organization in his book, At Home in the Universe, (1995, New York: Oxford University Press). In his example, the state consists of a two-dimensional array of light bulbs (elements) that each have two possible states - *on* or *off*. The elements of the state have a higher ordering, from the outset, because each has binary potential. Elements in a "classical space", such as represented by the array, always have, as a minimum, binary potential, because each necessarily has "sidedness" In this case two sides.

The state-space for the collection is the listing of all possible arrangements for the sidedness of the elements as a set or space. Consequently, the potential of the system to display an overall structure is larger than just the number of elements - it is not static as just one expression. This potential is represented by a power function (discussed below). If there is no feature of self-organization, between the elements for their individual *on*/*off* condition, then the number of possible states the entire system can display is at a maximum.

The rule for this maximal state-space is that if there are (n) elements, then the state-space has $(2)^n$ possible arrangements. Thus, if the number of elements in the array is 16, the number of possible states in the state-space is $2^{16} = 65{,}536$. In this framework there is no basis for self-organization.

The mechanism that creates a self-organized structure is linkage in a Boolean network. The condition of each element depends on the state of two other elements by arbitrarily applying a Boolean rule. The relationship of the two linked elements (for example, that one is *off* and the

other *on*) is transmitted, on each iteration, to the single element to which they are linked. This establishes whether the single element will be either *on* or *off* on the next iteration in time. A dynamic cycle of all the possible states is initiated and displayed through successive iterations of the state-space.

The function that establishes the number of possible states in the above state-space is $n^{1/2}$. For 16 nodes, there are now $16^{1/2} = 4$ possible states in the system, not 65,536! Thus, there is a dramatic reduction in the number of possible states the state-space can display. The collection of all possible states of the system translates, in quantum terms, to the collection of "histories" that contribute to the one overall "observed" state in each iteration. Of course, the iterations, in this case, are not simultaneous (as they would be in a quantum framework). Rather, they are sequenced in "time" because it is a classical construction.

There is very large relative increase in the order of the system as the number of elements increases. In other words, the number of histories does not keep up with the with the increase in the number of elements in the total state-space. Factiously, we might even state that, in relative terms, for an "infinite" system of elements, the number of possible, observed events will be so small that it approaches unity.

Table 4: Comparison of 16 and 25 Elements

	16 Elements		25 Elements	
	Total History (State-Space)		Total History (State-Space)	
Non-linked	2^{16}	65,536	2^{25}	33,554,432
Linked	$16^{1/2}$	4	$25^{1/2}$	5

What is the general framework under which self-organization operates?
The phenomenon of self-organization takes many forms in Nature. Wikipedia has the following statement on diversity of examples: "As the list grows, it becomes increasingly difficult to determine whether these phenomena are all fundamentally the same process, or the same label applied to several different processes. Self-organization, despite its intuitive simplicity as a concept, has proven notoriously difficult to define and pin down formally or mathematically, and it is entirely possible that any precise definition might not include all the phenomena to which the label has been applied."

To give some clarity to this issue, the same features identified in EPR structures can be identified in a general framework for self-organization. Both formats (frameworks - quantum for EPR phenomena and classical for self-organization structures) each have two dimensional platforms as their basis.
The difference is:
For EPR structures, the first platform is a lower-dimensional, quantum structure that is not open to observation; and a second is a classical platform that is open to observation.

For self-organizing structures, both of its platforms have a exclusively classical bases. Each element displays dynamic "sidedness" (in time) from the outset. This is a purely classical feature. For comparison, the individual elements in a quantum state are non-dynamic (they cannot display overall evolution in time.)

The commonality is:
For both EPR and self-organizing structures, dimensional levels determine what is observable and what is not. In each case a not-set construction is created that can be identified through careful analysis of the relative dimensional operations in both types of structure.

The reason each phenomenon of self-organization has a "unique signature" (that cannot be mathematically generalized) is that the phenomena have no common properties. The only common factor is the framework of "not-set" construction. In this format, elements that share common property are paradoxically conjoined. This contradiction produces a state in which the parts are not separably and structurally observable, in spite of the fact that they form a contiguous whole for the property they share.

The half-silvered mirror experiment as a template
The half-silvered mirror experiment is the simplest EPR example of a state that contains two parts conjoined paradoxically (one quantum-mechanical and the other classical). A choice is required for that which is observable. Whichever choice is made the other perspective is hidden.

For the quantum framework - parts that would otherwise have classical identity are concatenated through a device that unitarily links them. The parts are then entangled, and do not evolve in time. The reason they do not evolve is that their dimensional framework cannot support separate elements in time. As described in the discussion on EPR phenomena, time is, rather, assigned to a direction in the structure that is imaginary. Accordingly, in mathematical description, the factor *i* is attached.

For the classical framework - the parts that were entangled in the quantum format are observationally separable because the dimensional structure has been raised, which allows time to take on separate properties in the space. Consequently the separable parts display sequence in time.

Comparison to the structure of self-organized frameworks
For the linking of two paths, in the half-silvered mirror experiment, and elements, in the Boolean relationship of the array, what would otherwise be distinct in a purely classical format and have no basis of linkage, is entangled. The result of this entanglement, in both cases, is that, a choice is required for what is observed. The concatenated elements form a non-observed dualism that produces a single state. When the state is disturbed, at the quantum level, it collapses and the individual paths become real and observed.

Because the structure of the array is purely classical, the choice that applies in terms of

observation, is different but points to the same principle. Specifically, the framework of choice is that what is observed is either the linkage of elements, or the iteration they produce. The reason both are not observable simultaneously is that the elements that are linked are precursors, in time, to the single state that is the observed event. This is a spontaneous sequencing between an entangled relationship and the state it produces. As for the EPR experiment, we cannot observed both simultaneously. In comparative terms, the framework of the transformation in the EPR structure is to square the modulus. The framework of the transformation, in the example of the array, is action in time. The difference is entirely based the dimensionality found in each type of structure.

The role of the bifurcation of space an time

In the half-silvered mirror experiment, the outcome, for each photon that crosses the state, is unitary - on exit from the structure, only one path (|F>) contributes to the expression of the state. However, in the classical format both (|F>) and (|G>) contribute. What required only one event for its unitary framework, then requires two events for expression of the total potential of the state. Observationally, a single classical event is not the whole of the potential for the state.

In descriptive terms, this is equivalent to a process of self-organization. Observationally, the potential of the state (as not organized) is larger that the "organized" form which is observed. As for the process of self-organization, there is a contraction in what is observed from the maximal potential of the state.

This analogy has application in the two dimensional model described in section .... The relative rates of accumulation for space and time were analyzed for the iterations of the infinite series based on the functions n^2 (for space) and 2^n (for time). The two infinite series produced do not accumulate at the same rate. The accumulation for space (as the summation for the cycles) is "left behind" for the accumulation of time.

As for the analogy in the example of the half-silvered mirror experiment, the total potential of the state as space and time is successively contracted in terms of space only. The potential of the state is greater than the potential that is displayed in terms of space. We can speculate that this is equivalent to linking elements in the Boolean array. If this analogy holds, then it points to the mechanism responsible for the spontaneous self-organizing force that produces the real features of the universe.

Can Mathematical Representation Be the Root of Reality?

A mathematical system is a structure of numerical elements and operations on them. Operations allow the grouping of elements of the system to represent complex relationships that mirror physical reality. All such systems operate within the limit of the abstraction called "infinity".

Infinity is not a number unless it refers to the limit of a bound system that converges to a specific value. In physics, although calculations may involve infinities, the final result of any calculation is not permitted to produce an infinity. If an infinity is the end product of a calculation then the

mathematical system has broken down.

A Very Big If

By definition, the "universe" is an infinite system. No reference is possible to any "outside" realm as the basis of its ordered structure. If part existed outside then we would simply have defined its boundary incorrectly.

However, there is another way to look at this. There is a framework of relationship in which an "inside" and "outside" to the universe exists that is not simply based on a faulty understanding of the "inside". The two regions are fundamentally paradoxical. By definition, no possible relationship between them can exist. Then, the boundary to the universe that we can explain, whatever its form, remains intact both observationally and for our theories, and we acknowledge that the theories and logic of rationalism and mathematics are not designed to provide final answers.

Indeed, it seems that this is exactly the situation. In spite of our great advances in understanding the universe, and infinity, in theoretic terms, we are not able to resolve the paradoxes that arise at the boundary of structure that is "real". In the theoretic arguments of philosophy and logic, we have had limited success in resolving the issues surrounding such presence of paradox but only through arbitrarily limiting the frames of reference that are applied for logical systems.

The region before the instant the universe came into existence cannot be explained in terms of the period after that instant. Equally, our current theories of the universe account for only a very small part of its potential. We have no understanding for the ninety-six percent that comprises dark matter and dark energy.

The thesis of this writing is that the universe does not evolve in the same format as is represented formally in our logical systems. Rather, the universe evolves its extension and structure of relationships in a process of self-organization. In this case, the laws develop sequentially, and require no outside guidance for the sequence. There is no reason to exclude self-organization as a framework for the creation of the universe, because self-organization within the universe is a well-documented phenomenon.

The justification has been extensively discussed that the mechanism responsible for the process of self-organization is the paradoxical conjoining of elements in a structure. If the universe is self-organizing, then paradox is also a fundamental mechanism for that process. In this case, our mathematical understanding will always fail because, at the limit of the small and large scales for the universe, all mathematical systems meet a boundary across which the structures are paradoxical.

The fixed as pre-existent systems of our logic, and the process of the development are not the same. The only mathematical system that could completely represent the basis of the universe would be one that evolves its form, and there is no such basis for a mathematical structure.

i

The function *i* is the basis of the fundamental paradox between quantum and classical formalism. What is sequenced in space and time becomes simultaneously and entangled. This is found in all EPR experiments. The half-silvered mirror experiment is the simplest example. A flipping of the relationship of paths occurs such that the frameworks of action cannot be members of themselves even though they refer to the same structure.

The classical format is open to us, as observers, because we co-exist in a consistent framework of dimensionality that permits real extension for space and time. By contrast, the quantum format is closed to us, as observers, because there is no such extension displayed. Each of these formats is an incomplete expression of the totality of the two. They mirror the dualism that is the *footprint* of the native universe - that two parts are conjoined paradoxically in a single structure.

The debate on whether classical reality and its formalism, classical, relativistic mechanics, can correct represent the nature of the universe is largely over. Bell's theorem ended the discussion in a single sweep by (supposedly) showing that the classical picture is wrong in its own basis. However, Bell's theorem is based on the wrong question. It does not account for the fundamental role of paradox in creating two equal descriptions of the universe - one quantum and the other classical. Together they represent the native universe that has as its basis, immaculate non-order.

Linguistics and Mathematics

A paradoxical relationship is found between frameworks of construction for linguistics and mathematics. It is based in the difference between the format of symbology across them. The relationship of the "noun" and the "natural number" exemplify this. Nouns are not fungible identifiers (not exchangeable across contexts of identity). Examples are "dog" and "cat" where "dog" represents only "dog" and not "cat". The natural numbers are fungible (for example, 1, 2). "One" represents one of anything.

Because the linguistic symbol refers to a single and discrete item, it allows representation and recounting of non-general (non-fungible) actions - for example storytelling. For the mathematical identifier, discreteness is prohibited and its identifiers are entirely general. This allows the mathematical object to be universal in its reference, and it is why mathematics is able to form the basis of physical theory that is universally transformable across specific contexts.

A fundamental element of paradox applies across the difference between "what is not fungible" and "what is fungible". Neither is a basis of the other, yet together, they are paradoxical constructions for describing the same system which, in the largest sense, is the universe. This paradoxical relationship, is another *footprint* of the fundamental dualism that is the property of the "native universe" from which they arise.

The Philosophical Relationship between Paradox and Logical Proof

The argument that paradox is a fundamental feature of the universe does not rest on logical proof. In fact, it cannot. This is because logical arguments cannot embrace paradox as a mechanism. For

logic, the discovery of deeper truth between dualistic elements, requires that some basis of logical transformation apply. A deeper linkage has then been discovered.

By contrast, in demonstration that related dualistic elements are conjoined paradoxically, any logical transformation is categorically prohibited. Paradox from the outset prevents this. Thus the only tool for understanding this mechanism is comparative analysis. The strength of the argument then rests on showing that the same role of paradox operates across fundamental dualism in widely diverse examples.

We should expect the universe to display a general, common framework of structure in any manner, format and subject by which we open it to observation, be this in linguistic terms, mathematical theory or physical experience. The universe will give the same information to its fundamental nature. It should also be expected that the form of expression will be unique in each case. This is the only way the universe can take on its diversity of forms. In fact diversity itself has, as its mechanism, paradox without which the uniqueness represented in all diversity would collapse.

Logical proof has a role to play within each such element. However, it has no role to play in the analysis of the uniqueness across the parts of paradoxical dualism. In that case we have broached some limit of the infinity.

The following sections contain further examples of the role of paradox in the creation of ordered systems. Most of the examples are far removed from the realm of pure mathematical theory and physics but point to the same conclusion. Their commonality is that they all mirror the structure of the "native universe". In each case, when we open the universe to observation and attempt to draw singular conclusion, a dualism of possible conclusions arises. Each part is paradoxically conjoined to the other.

Religion and Science

In the most general sense, all religions are based on the certainty that the universe is not knowable in its ultimate form. This is the realm of "god". In paradoxical association, "ultimate" science, is based on the belief that the nature of the universe is knowable through discovery of a single fundamental physical principle.

We find two paradoxically conjoined observational perspectives for the same fundamental structure, the universe. This is another *footprint* for the ultimate form the universe requires to exist in the first place. The mechanism of paradox is always the flipping of relationship of the elements that form a common state such that they are not members of themselves. In the case of the relationship between science and religion, there is a paradoxical "flipping" across the first assumptions regarding ultimate understanding. For religion, there is an ultimate prohibition on understanding, and for science there is no such basis for prohibition.

Equally, we see that "certainty" (through a belief system) is the operative framework for religion,

and uncertainty (through the need for proof, and belief in the possibility of finding it) is the operative framework for science. This means that, true to all paradoxical constructions, these two correlated, but categorically separate, frameworks of understanding entangle the elements that are the property of each perspective. Each includes the fundamental aspects of belief, knowledge, certainty and uncertainty in its basis. The context of how these terms apply is simply flipped, and this is the hallmark for how paradox conjoins elements as dualisms that are non-resolvable across their boundaries. Science and religion are paradoxical correlated constructions for the same system, and this is the format of the Russell set.

If this interpretation is indeed the final meaning we can take for what is infinite in the universe, then there is no single framework to understanding its ultimate form. Paradox will always fracture this understanding. Furthermore, because of the operative mechanism of paradox, neither basis can be successfully used to disprove the other. What we are left with is a the universe that is immaculately non-ordered. Out of this non-order a force and trajectory creates a self-organized reality that mirrors its origin - the immaculately non-ordered universe.

There is no conclusion on whether a god or science is the source of fundamental understanding on the complexity of the universe. If a choice is required then it can only be based on faith. The best we can do in understanding and exerting influence over an immaculately non-ordered universe is to manage its chaos.

Literalism, Humour and Quantum Mechanics

Humour can be thought of as the quantum mechanics of linguistics. The basis of construction for each is that two paradoxical parts are joined as members of a common state. These parts are not members of themselves for the state to which they belong and this is the defining feature of the paradox found in the Russell set.

Humour is commonly defined as, "the juxtaposition of the incongruous". Story telling that is exclusively humorous (other than that is also part of some larger some literal structure) does not extend. The mathematical equivalent of such non-extension is the quantum-mechanical description, and, as for the quantum description, the reason that humour does not extend is that the argument is self-circular.

An example

"What did the Zen master ask of the hot dog vendor?" Answer: "make me one with everything." The self-circularity is that the phrase, "make me one with everything" refers to two frameworks that have nothing in common, yet refer to the same state. They are juxtaposed and incongruous. The common element (that the Zen master asks a single question) is the common universal structure that internally contains a juxtaposition of two statements and meanings. The two statements are entirely self-circular, since neither refers to the other in logical terms, and we cannot resolve which meaning should apply. We are forced into an endless loop as to which statement is true (which singular meaning is intended), for the common framework that the Zen master has asked a single question.

The reason we laugh at humorous stories goes back to our innate perception that there must always be some final and singular meaning to events and, in general, causality. Humour proves this wrong, and our only possible response is the expression of (nervous) pleasure at the strange difference between these two fundamental forms for causality.

It is also instructive to compare literal and humourous descriptions. For story telling, literal recounting has extension both within and beyond the content of the story. In other words, the story has an ending that can be extended by new events and expanded for the events within. The structure is without limit, in agreement with our classical experience. The linkage of elements is rationally sequenced for our classical experience of cause and effect. This is not the case for humour. For humour, the structure of the argument is closed both to logic and extension. The framework is "infinitely contained".

Both formats (for humour and literalism) must give up something for their complementary forms. Literal story telling gives up finality as endings are alway arbitrary. Humour gives up consistency because we are not permitted to draw conclusion on meaning.

The Comparative Veracities of Quantum and Classical Theories

In 1994 a debate was held between Roger Penrose and Stephen Hawking at the Isaac Newton Institute for Mathematical Sciences at the University of Cambridge
[Stephen Hawking and Roger Penrose. 1995. The Nature of Space and Time, Princeton: Princeton University Press.]
Some excerpts from the lecture are given below which support the contention that the classical and quantum pictures of the universe are equally correct in their own frameworks. They paradoxical formats for descriptions of the same infinity

Foreword by Michael Atiyah: "Quantum theory, or its more sophisticated version, quantum field theory , is now highly developed and technically successful, even if there are still philosophical skeptics such as Roger Penrose. General relativity, Einstein's theory of gravity, has equally stood the test of time and can claim remarkable successes, although there are serious problems concerning the role of singularities or black holes.

The real issue that dominates the Hawking-Penrose discussion is the combination of these two successful theories, how to produce a theory of "quantum gravity".

Hawking on Classical Theory p. 4: "General relativity is a beautiful theory that agrees with every observation that has been made."
Penrose on Quantum Theory p.61: "It has been said that quantum field theory is the most accurate physical theory ever, being accurate to about one part in 10^11. However, I would like to point out that general relativity has, in a certain clear sense, now been tested to be correct to one part in 10^14 (and this accuracy has apparently been limited merely by the accuracy of clocks on earth)."

The question as to whether classical (relativistic) or quantum theory is the correct final basis for

the description of the universe is the wrong question. The correct question is, "why are there two absolutely correct and final descriptions that are paradoxically composed?" The answer is that the native universe is composed as a dualism of paradoxically conjoined parts, and these formalisms, as a dualism, are one *footprint* of its form.

Some Speculative, Circumstantial Evidence

From the Cosmic Background Explorer Satellite

The waves in the cosmic fireball should appear randomly around the sky at all sizes. But, according to the new map, there seems to be a limit to the size of the waves, with none extending more than 60 degrees across the sky.

Reference:

http://www.nytimes.com/2003/03/11/science/universe-as-doughnut-new-data-new-debate.html?pagewanted=2&src=pm

The interpretation of this phenomenon is that it points to the hexa-orthogonal precursor to the emergence of classical space. The hexa-orthogonal basis of sub-classical space is described in the section, The Geometry and Mathematics of Sub-classical Space (p. 34).

The structure of an hexa-orthogonal basis also speculatively explains the instantaneous faster-than-light expansion of the classical universe in a new way. A flat plane basis that is hexa-orthogonal has more directions defining its space than the classical basis which, by definition has only four. If the six-sided basis were to collapse into a four-sided basis, there would be too many locations in the new, simpler space. A faster-than-light precipitation would occur as the entire structure emerged to its classical form. This simultaneous collapse of the entire structure would also result in a uniform distribution of matter, as is observed.

Quark Structure

The hexa-orthogonal geometry has significance to account of the "strange color force" or "color confinement" of quark structure. For the grouping of the six fundamental quarks the force between them is unlike any other in that it becomes stronger, the farther individual quarks are pulled apart from their grouping, and single quarks have never been observed.

This force can be interpreted, under the geometry of the model. The grouping of six quarks represents a fundamentally different basis of orthogonality that has six defining directions in its space. As such, there are too many locations defined in its structure relative to classical space. When quarks are pulled apart, we are attempting to collapse a basis of six directions into a basis of four, and there will be an increasing pressure as the extra locations, defined by the dimensionality of the hexa-orthogonal basis, are forced into the smaller classical space.

References: 1. Bloom, Elliott and Feldman, Gary J. 1982. 'Quarkonium', Scientific American May 1982: 66-77.

2. http://en.wikipedia.org/wiki/Color_confinement

Other Topics

There is no limit to the number of topics in which the *footprint* of the native universe can be seen at work. They range widely including mathematics, physical science, social science and the arts. This is to be expected. The largest property of the native universe must be mirrored in all the forms contained within it.

The Relationship Between Art and Science

The relationship between art and science is paradoxical for the format in which each is created. Science has as its basis the creation of general principles that extend across examples taken from nature. By contrast, artistic creations are specific only to the subject of each work. There is a paradoxical flipping between "general" and "specific" representation.

Rejection, Tolerance, Acceptance

Rigorous societal values are statements of universal principles no less that any attempt to define the nature of the universe through physics. A given social or religious understanding is taken to represent an ultimate truth. Once again paradox intervenes, and we find that such visions of absolute truth break down. As a minimum, two complementary views emerge spontaneously, and there is no basis of commonality between them. This can lead societies into conflict that is not resolvable.

The fundamental basis of paradox is the "prohibition to knowledge". As for Heisenberg's uncertainty principle, the "sliding scale", in a social setting, across paradoxical states, is "rejection", "tolerance" and "acceptance". Of course, at the worst, if "rejection" is the basis of knowledge, it can also be the justification of violence. At the best, if "acceptance" is the basis of knowledge coexistence is possible.

Conclusion

In the final analysis, for all our observations and conclusions in what we observe of the universe (our knowledge of the universe), there are no absolute truths. The universe, in its largest state, is immaculately non-ordered. The unique structures that, in composite, make up the larger fundamental state of the universe, must each consistently reflect the larger state in which they share common property. For intellectual study we separate these parts into manageable topics of philosophy, pure mathematics, physics, chemistry, religion, politics, and social interaction, as a few main examples. Regardless of which area we study, these parts must reflect a common mechanism to the universe.

The basis of the theory in this writing is that paradox is the mechanism that creates non-reducible dualism in all fundamental structures. The provisional proof for this is found in the many individual examples that support this conclusion and the absence of counter-examples. If all the fundamental parts of the universe have as their basis the mechanism of paradox, then the universe in the largest sense must itself display paradox.

On the surface, it would seem that a fundamentally paradoxical universe would not be able to

develop the complex structures we know it has. However, this is not necessarily so. If there is a mechanism for hiding paradox then "form" is possible. In physical structures the mechanism of "hiding", is dimensional bifurcation. Equally, in theoretic arguments it shows up "statically" in two ways - as incompleteness and inconsistency.

The term "fundamental truth" refers to any conclusion we derive and consider to be a final and complete understanding. For any given topic, it is an understanding of truth that is universal. However, any such conclusion must reflect the general property of the universe that the fundamental mechanism behind all "form" is paradox. The two forms of this are that our conclusions are always necessarily either incomplete or inconsistent.

Acknowledgement

I wish to thank Eric Kvaalen for his valuable comments on my interpretation of Hardy's paradox.